1946

Investigation of amplidyne position and rate-controlled power drives

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http://hdl.handle.net/10945/6434

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CAMBRIDGE 39, MASSACHUSETTS

September 16, 1946

Captain Buracker
Room 5-233
M. I. T.

Dear Captain Buracker:

Subject: Thesis Accomplishments of
W. A. Hasler and T. R. Weschler

I am pleased to inform you that I regard the thesis program conducted during this past summer term by Messrs. Hasler and Weschler as highly successful. The work which these men undertook leaned heavily upon the subjects that they had studied during the previous two terms and also it necessitated considerable resourcefulness and perseverance on their part.

Their document is a comprehensive one and well-written. In spite of the fact they were delayed in starting their work because of the late arrival of apparatus being loaned to the Laboratory by the Navy, they accomplished all that I had expected of them. I am very pleased with the results.

Very truly yours,

/s/ Gordon S. Brown

Gordon S. Brown

GSB:maf

INVESTIGATION OF AMPLIDYNE POSITION

AND RATE-CONTROLLED POWER DRIVES

by

William Augustus Hasler, Jr.
S.B., United States Naval Academy
1938

Thomas Robert Weschler
S.B., United States Naval Academy
1939

Submitted in Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Electrical Engineering

at the

Massachusetts Institute of Technology

1946

ACKNOWLEDGMENT

The authors wish to express their appreciation to Dr. Gordon S. Brown and the staff of the Servomechanisms Laboratory for their advice and assistance with the problems of this thesis and for the loan of the Laboratory facilities. In particular, thanks are due to Mr. D. D. Terwilliger for thesis evaluation and to Mr. Paul T. Travers for aid in applying the decibel-log frequency approach to this problem.

TABLE OF CONTENTS

CHAPTER I

INTRODUCTION

World War II ended with the scientific advances in fire control--whether of rockets, bombs, or guns--leading to the catch phrase "push-button war." Though overpopularized and often misunderstood, this phrase indicates a sound military factor; that the elimination of the human element in a functional position improves performance.

From the standpoint of shipboard installations, the greatest requirement to date is that each gun must comprise within its mount a complete control system. This requirement is dictated by consideration of simultaneous attacks and is extended by the "push-button" motive to include the elimination of human operators wherever possible.

The fire control problem includes target acquisition, tracking, determination of lead angle and trajectory corrections, and correction to errors in the solution. Acquisition and tracking are initially radar problems, the movement of the gun to match the indicated track a servomechanisms problem. The determination of lead angle and corrections lies in the field of computer design. Servomechanisms must again translate indicated angles into actual gun positions. The errors in the solution may be determined by human eye, radar, or radio, but a servomechanism will be needed to introduce the correction to gun angle. Thus it is seen that servomechanisms are mechanical substitutes for human operators as well as sources of power.

For shipboard use the track is made much more difficult by reason of the instability of the gun platform. For most guns the maximum

accelerations and velocities are demanded to meet the roll and pitch of the ship rather than to follow high speed and maneuverable targets. Simple systems capable of detecting and removing ship movement at the gun, and also capable of acting on tracking information as accurately as before were desired. The rate-controlled servo offered the smaller amount of equipment for the better job when compared to the position-controlled servo.

Simultaneously with this demand for improvement in the servo performance there developed within the engineering field an increased understanding of the servomechanism technique. The industry was ready when the need for rate servos was recognized.

As this has been a recent development, there is considerable interest in the problem of converting position-controlled to rate-controlled servos. This thesis proposes to explore this problem in its particular application to gun power drives having amplidynes as their prime movers. Representative figures for amplidyne and gun characteristics will be used as necessary or desirable to illustrate material, to allow selection of a course of action, and to represent quantitatively mathematical concepts.

CHAPTER II

BASIC THEORY OF

POSITION-CONTROLLED AND RATE-CONTROLLED POWER DRIVES

Essentially all power drives are motors. Hence for a constant signal, velocity results. This signal may be supplied by any device. For this problem, the devices concerned will be position and velocity error-measuring means which are described in some detail in Chapter III. Discussion is restricted to simple proportional servos (type I) unless specifically stated otherwise.*

If the signal is produced by an error in position, then when the driven member moves to the point coincident with the control member, the error becomes zero and the driven member stops. Hence a position servo using a motor is essentially a zero positional error device, and for a fixed control input reaches this zero. If the position of the control member is changing at a constant rate, then the error increases until it has reached a great enough value to cause the driven member to move with sufficient speed to match the speed of the control element. These statements disregard transient effects. Thus the error in a position servo, when the input is a velocity, is a measure of the gain of the system. With a high gain, this error may be kept small. If the system is a slow one, that is, has an appreciable time lag between change in input velocity and change in output response, the error may, during this transient phase, reach much larger values; but once steady state exists, the error returns to the value determined by the system's gain.

A rate servo compares velocities to determine error. Hence the

*Type I Servos as defined in the paper, "Transient Behavior and Design of Servomechanisms," 1943, M.I.T., Prof. Gordon S. Brown, Sec. 5, P. 10.

output velocity must always be just sufficiently less than the input velocity so that the existing error will produce this output velocity.

A position servo operating at constant velocity introduces a constant positional error to maintain the approximate match. A rate servo operating at constant velocity introduces a constant velocity error to maintain the approximate match. Thus, over a period of time, the position servo's positional error does not change; but the rate servo's positional error increases constantly as the integral of its velocity error. Strictly speaking, a rate servo, dealing only with velocities, has no position in its loop. To speak of position error for a rate servo it is necessary to think of it as being in some application, such as in the stabilization loop of a gun (see Chapter III). Then position error may be reckoned as the difference between the initial position of the gun barrel with respect to the stabilizing gyro's output axis and the final position of the gun barrel with respect to this same reference.

As with positional error in a position servo, the velocity error in a rate servo for any particular output velocity is a measure of the gain of the system, and with high gain this error may be kept small. However, any time lag in a rate servo in reacting to a velocity change introduces an increased positional error by virtue of the integration of the velocity error over this period of transient response. Hence a rate servo must have both small time lags and high gain to insure small positional and velocity errors.

It is evident that a rate-controlled servo cannot be used for stabilization without some external element since there is no means of

detecting and correcting positional error. All fire control uses of power drives include either a manual or automatic tracking device which provides this control. A position servo can function alone to provide stabilization. This constitutes no drawback to the rate servo in its fire control application, however, since the problem of tracking is one of staying on target. So long as the effect of ship roll is almost entirely removed, tracking errors develop from variation in target motion, variation in ship motion in the plane of the earth's surface, and human or auto-tracking deficiencies.

What advantages exist at present for the rate servo? The answer will be given in two parts; first as applied to the rate servo alone, and second, as applied to the rate servo as the power drive in a shipboard gun installation locally controlled.

From a mathematical point of view, a rate servo is inherently more stable than the position servo from which it is adapted. This may be demonstrated by multiplying any position servo's frequency response by $j\omega$. A comparison of the two plots will show that the rate servo has a smaller peak in its frequency response and a higher resonant frequency.

From a practical standpoint, position servos depend for positional information primarily on synchros (see Chapter III). All present Navy synchros are 60 cycle AC devices. Rate servos are not so restricted. The choice of frequency has been 400 cycles, and all the attendant advantages of this type of power supply are available in the electronic sections of the servo loop.

Considering the rate servo in its application as the power drive

of a locally controlled gun mount, this system requires fewer and smaller components. Stabilization information is provided by two small rate gyros in contrast to the heavy stable vertical and attendant control elements associated with a position system. If a disturbed line of sight computing gunsight* is employed (the present trend) no separate rate gyros are needed since those in this sight may serve the dual function of computing and providing the stabilizing information. Tracking information is provided by a simple potentiometer handlebar arrangement in the rate system. The position system requires two channels of synchro information (low and high speed) for accuracy. Because of the smallness of the handlebar and sight arrangement for the rate system, the unit may be mounted on the gun. The position system has usually had an external director for each gun, making the alignment problem more difficult, introducing parallax errors, and occupying more space.

As disadvantages, it has been pointed out that the rate servo, considered in a stabilization loop, requires an external positioning means which the position servo has as its basic feature. Also, the rate servo has a greater tendency to instability at high frequencies. This is borne out by the same mathematical approach that proved the rate servo to be basically more stable. Multiplying a position servo's transfer locus by "j" gives a locus exhibiting more stability at all frequencies. The corresponding rate servo's transfer function is given poor characteristics at high frequencies by the "ω" part of the multiplier $j\omega$. This factor becomes larger and larger carrying the locus more closely by the $-1 + j0$ point, the basis for stability determination.

*A-1 Computing Gunsight (Restricted), 1946, U. S. Army - Dr. C. S. Draper

One other factor which must be considered in servo synthesis is torque demands. The torque loading on a gun mount is not constant. The friction load varies with temperature and maintenance conditions. The transient load, apart from inertia and elastance considerations, varies with conditions such as the unbalance of the gun about its center of rotation coupled with ship movement or the firing of a wing gun in a multiple gun mount.

In a position servo, torque increases will slow the motor momentarily, but the gain will allow proper speed to be restored at the cost of a small increase in positional error. The transients involved will have no effect on error once steady state is reached under the new loading, since in a position servo no permanent positional error can result from transient action.

In a rate servo, torque increases will likewise slow the motor. At high speeds the rate servo will respond quickly to the new load at the expense of output velocity, thus increasing the velocity error slightly. The transients involved will produce a positional error as the integral of the velocity errors during the transient period. These conditions might be acceptable as they stand. However, the action of the rate servo at low speeds is the controlling factor. If the input speed is very slow it is not only possible but quite probable that a heavy torque load applied at the output will stall the motor. (This does not apply to motors having flat or rising torque-speed characteristics.) This condition can exist, speaking of amplidynes, whenever the increase in velocity error as the result of stopping the output produces a control field current less than that needed to excite the breakaway

current for the torque load involved. No increase in positional error will remedy the situation until the external tracking element comes into play. It is desirable to reduce the reliance on this element to a minimum in the fire control application. Hence the power motor in a rate servo should be typified by a rising torque-speed characteristic. The condition will hereafter be referred to as torque-compensated.

Torque compensation introduces one further consideration. Such compounding is achieved by some type of positive feedback, either internal or external. Positive feedback is inherently unstable. Therefore, the method of compensation employed must not lead to resonant peaks in the power supply at low frequencies which lie within the normal operating range of the servo.

Having determined the requirements of a rate servo and specified how its power drive must perform in the simplest servo loop, it is necessary to examine the feedback elements and corrective networks in a typical modern position servo used as a gun drive to investigate their applicability to use in a corresponding rate servo. The function of these elements and networks in the position servo will first be specified to aid in determination of their probable effect and usefulness in the rate servo.

The feedback elements in the position servo, other than the fundamental output feedback, are derivative devices capable of detecting a particular derivative of position, sometimes going as high as the third. Their chief purpose is to anticipate changes, that is, to have the gun already acting in the proper way to meet the positional change taking place by the time the positional error signal can convey this information

directly to the power drive. Even if the time lag in a derivative element is as great as that in the positional detecting component, the derivative element output augments the positional error signal to cause swifter response of the gun to the error change.

The corrective networks are essentially derivative or integral circuits. These circuits are selected empirically or mathematically through graphical analysis of uncorrected frequency response data. Their function is to improve the stability and performance of the system by allowing the loop to enclose a greater gain, by reducing the magnitude of the resonant peak, and by shifting this peak to a higher frequency. It is possible that the uncorrected response may be unstable in which case the primary purpose is to secure stability. A derivative circuit is generally termed a lead network since it introduces a positive phase shift in the response. An integral circuit may be called a lag network since its effect is to cause a negative phase shift in the response.

It is logical to assume that the rate servo will respond faster if the derivative elements are retained, rejecting possibly the element detecting velocity since this is the basic quantity in the rate system. Unless a ready means of detecting the next derivative after the highest one used in the positional loop is available, it seems best to close the rate loop with one less such element. This statement is based on considerations of simplicity and ease of conversion, especially avoiding the use of extra equipment.

The corrective networks, except by coincidence, must be different for the rate system. The rate system contains other and altered components and the basic quantity is velocity rather than position. In

fact, these corrective networks constitute the problem in the conversion. The experimental work done for this thesis has as its object the collection of sufficient data to allow a graphical synthesis of these networks.

To summarize, a rate-controlled servo for fire control use should be characterized by fast performance and high gain. There should be little phase shift between its input and output over the required frequency range. The power supply should be torque-compensated and, considered by itself as an open cycle system, must have no resonant peak near the operating frequency level. An external tracking device is fundamental in the system and serves to eliminate positional errors. This external tracking element should, so far as possible, play no integral part in the functioning of the rate servo as a stabilizing device, but rather should serve as a means of introducing target motion.

Hence it is evident that to convert a position-controlled servo to a rate-controlled servo the power drive is first to be torque-compensated; second, the open cycle frequency response of the power drive is to be obtained; third, the effect of the feedback elements must be added and the stabilizing networks selected through graphical or mathematical manipulation of this response; and finally, the position-controlled servo's components and networks are to be compared with those in the rate-controlled servo to facilitate change from one to the other.

The succeeding chapters follow this general program. Experimental work necessary to each step is presented in appendices. Results obtained from the test amplidyne are presented as typical of this form of power drive, and are used as the basis for general conclusions in

other chapters. The decibel-log frequency technique is employed to synthesize the simplest corrective networks providing both stability and performance. The effect of feedback elements has been omitted because of press of time. The final chapter evaluates the conversion problem in the light of experimental data and confirms or explains error in the basic theory presented.

CHAPTER III

TYPE SERVO LOOPS AND THEIR COMPONENTS

A. The Position Servo System

For any typical gun installation, the position servo system in simplified form may be represented as follows:

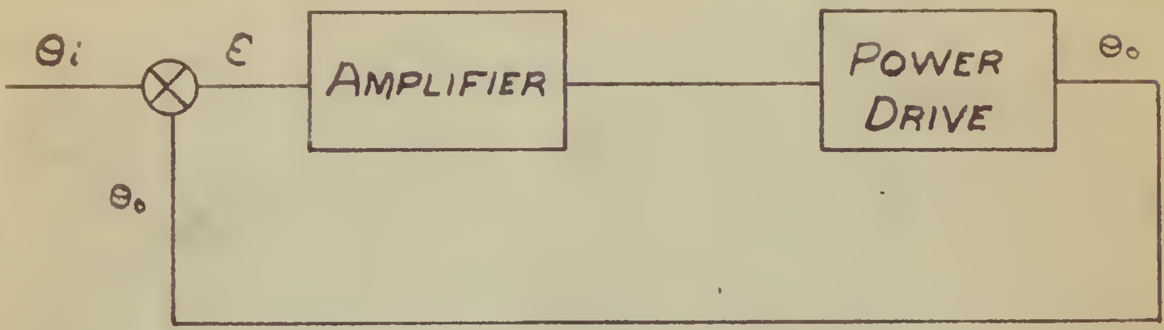


FIGURE 3-1

For purposes of better analysis of components so that a more definite tie-in with a rate system may be established, this system will be expanded to include a particular type of error-measuring mechanism and a particular power drive. These changes lead to the following servo loop:

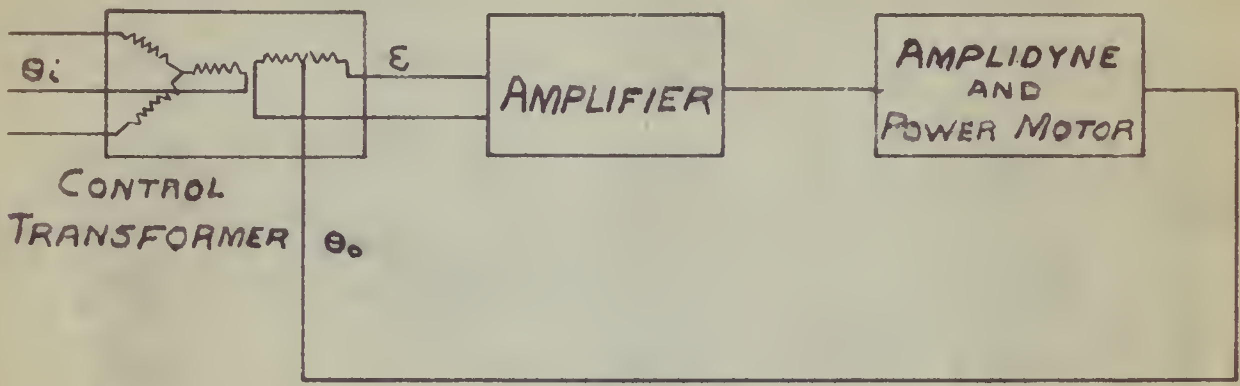


FIGURE 3-2

For purposes of stability, various feedback techniques other than the basic feedback of output may be required. This phase of the problem has been discussed generally in Chapter II.

B. The Rate-Controlled Servo System

The rate-controlled servo system is identical with the position system in its simplest form except that the input is an angular velocity (ω_i) and the output is an angular velocity (ω_o). Hence, it is apparent that the error-measuring device must differ from its position control counterpart. In addition, since the present technique of rate measurement employs devices mounted directly on the guns, the gun itself becomes a part of the servo loop. The single degree of freedom gyro has distinguished itself as a ready means of rate measurement. Since the torque output of a rate gyro is a function of input angular velocity, a torque pick-off attached to the rate gyro commends itself as a source of error voltage. A typical loop embodying these components is illustrated:

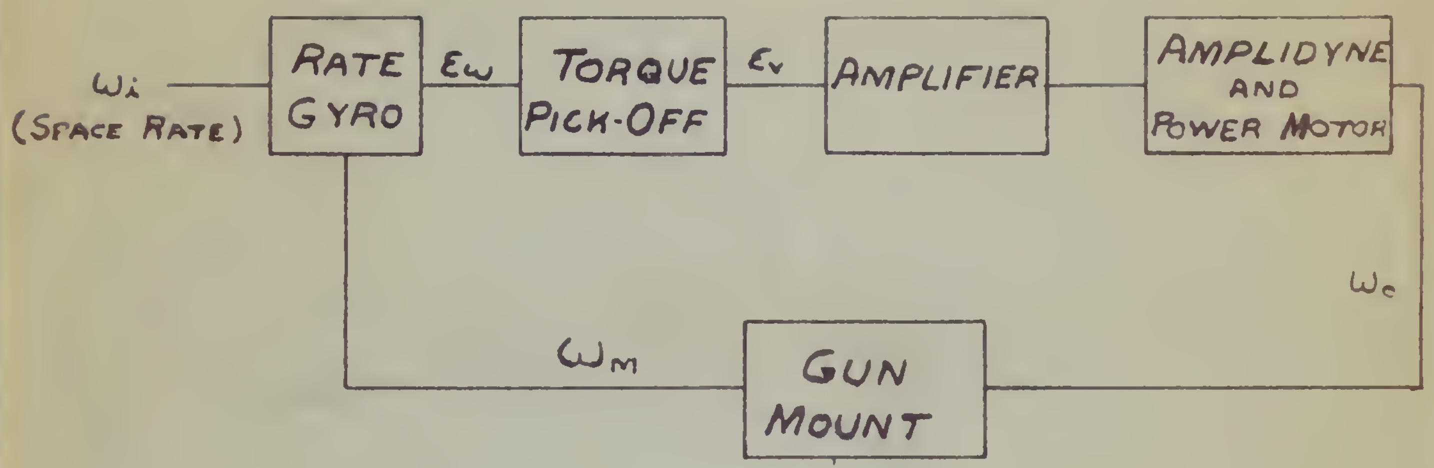


FIGURE 3-3

C. Components

1. Synchro Control Transformer

The synchro control transformer has become such a well-known device in fire control systems as to justify no extensive analysis of its functions. Basically it is an electro-mechanical differential having an electrical field input corresponding to desired gun position and a mechanical rotor input corresponding to actual gun position. The rotor windings provide an electrical output of error voltage which is a linear function of error in degrees over a small working range. The sensitivity of such devices using 60 cycle excitation may be approximated as T volts per degree of misalignment. The phase shift introduced by control transformers is negligible over the frequency spectrum in which use of such a device is contemplated. Hence the transfer function of a control transformer is $K_t G_t = T \frac{(\text{volts})}{(\text{degree})}$.

2. Amplifier

The amplifier used with a voltage input signal to drive an amplidyne is comparatively simple. It may consist of several stages of signal amplification and an output power stage using beam-follower tubes such as 6L6's, or it may conceivably omit the initial amplification if the error permissible in the system is large or amplification external to the amplifier is great. The amplifier must also include a phase-sensitive detector if the error signal is A.C. The control transformer and the torque pick-off require such detectors. In general, the amplifier is so fast that its time lags may be neglected. Thus its transfer function is $K_a G_a = P \frac{(\text{amperes})}{(\text{volt})}$.

3. Amplidyne and Power Motor

The amplidyne is essentially a power-driven magnetic amplifier supplying power to the output motor but susceptible to control by the input signal from the electronic amplifier preceding it. The amplidyne itself involves internal feedback, since it has a compounding field in order to improve its torque-speed characteristic. The series field which compounds with the externally excited control field is energized by the current in the direct axis of the amplidyne, which is the current flowing through the power motor. If it were possible to vary the number of turns in this compensating (series) field, a torque-speed relationship suitable to the particular application could be achieved. Typical curves are shown in Fig. 3-4.

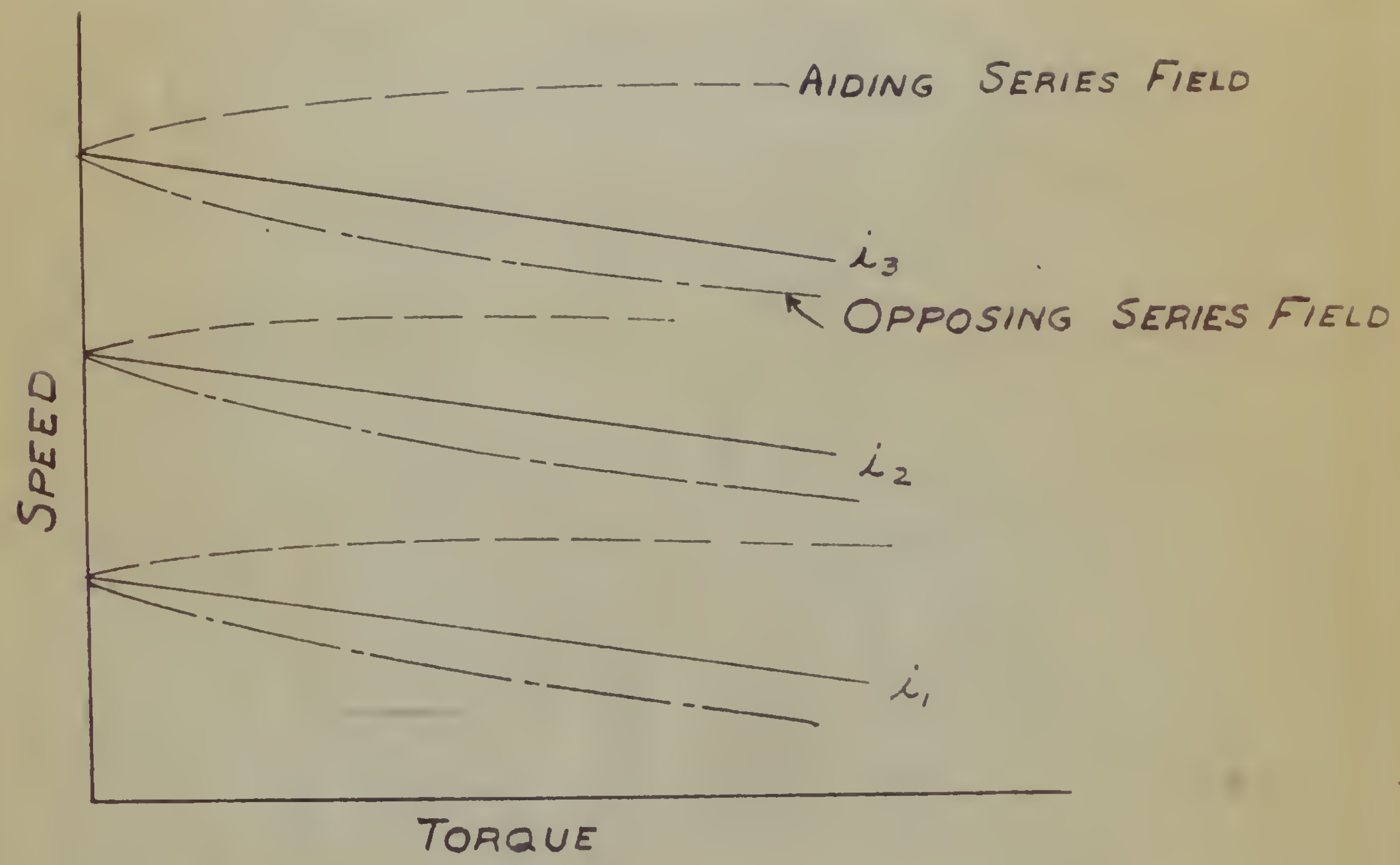


FIGURE 3-4.

By adjusting such a variable compensating winding it would be possible to cause the speed of the motor to remain constant or increase with an increase in load. The desirability of such adjustment is that the system will be "hard" or have an "active zero"; that is, it will have a restoring force immediately if some extra torque is imposed on the mount drive whether stopped or moving.

It is also necessary to insure that the amplidyne and power motor provide necessary torques at the speeds required of the gun. This must be checked against the accelerations expected and the inertia of the gun as reflected through the gearing to the motor shaft ($T = I\alpha$). Such data may be ascertained from mount characteristics and specifications to be met by the power drive. If the torque output is not satisfactory for this consideration, then the amplidyne set in use is too small and a change in the basic elements of the amplidyne is required.

With the fields properly matched and torque output satisfactory over the required speed range, a test of the amplidyne and drive motor as a single unit at various frequencies may be made. Such tests for amplidynes adjusted as described give transfer functions quite similar to those for two energy storage elements in series, thus:

$$K_d G_d = M(\text{rad/sec per ampere}) \frac{1}{1+2\zeta T_d S + T_d^2 S^2}$$

This is an empirical expression, since attempts to derive transfer functions analytically lead to 5th and 6th order equations involving constants not readily ascertained and requiring approximations concerning linearity. This empirical expression also disregards

7875

torque effects.

4. Rate Gyro

The rate gyro is a single degree of freedom gyro so mounted as to have its input axis parallel to the axis about which rates are to be measured. Since the rates with which fire control systems are concerned are those about the elevation axis (see Fig. 3-5) (line passed through the gun trunnions) and those about the traverse axis (line perpendicular to both the elevation axis and the center line of the gun bore), two rate gyros are required. It is to be noted that rotation about the gun line (center line of the gun bore) is disregarded since it has no effect on gun trajectory and hence on accuracy. With the rate gyros spinning at constant velocity (maintained so within .1% by the use of a frequency-regulated power supply) the angular momentum of the gyros along the axis of spin is constant, and will be represented as $H = I_g \omega_s^{a}$ where:

I_g = moment of inertia of gyro wheel

ω_s = angular velocity of spin

From gyro theory it may be stated that the instantaneous precessional torque about the output axis (see Fig. 3-5) is equal to the product of gyro angular momentum and the angular velocity of the gyro about the input axis, or $T = H\omega_{in}$. This involves a slight approximation; but, so long as the gyro is constrained to move only slightly from its neutral position, this approximation is so nearly correct as to be no measurable source of error. The transfer function of the gyro will be lumped with that of the torque pick-off, though if one is desired for the gyro alone, it may be represented as:

$$K_g G_g = H \frac{(\text{in lb. of torque})}{(\text{angular velocity in rad/sec})}$$

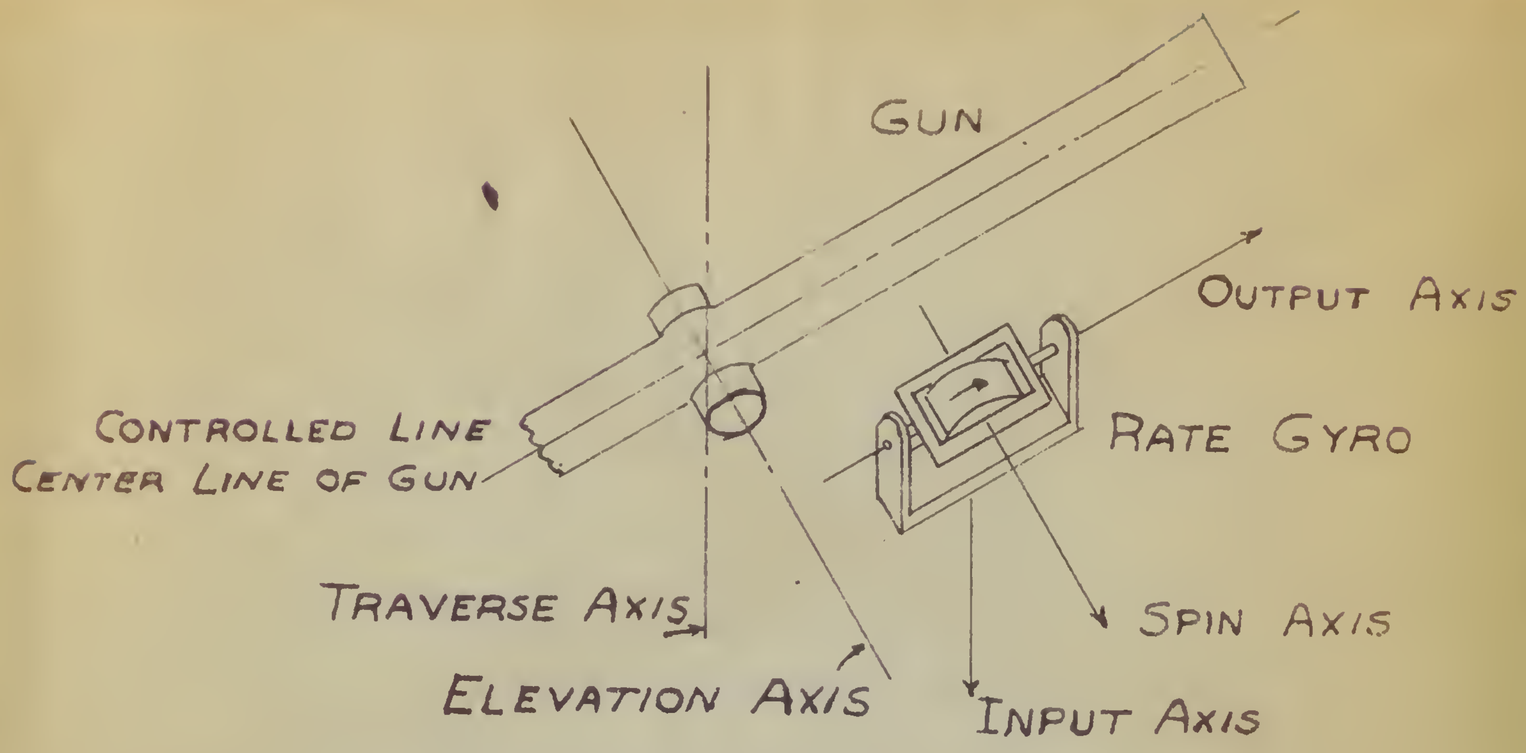


FIGURE 3-5

5. <u>Torque Pick-Off</u>

The torque pick-off is a device which reacts to the instantaneous torque of the rate gyro and by mechanical movement generates an electrical signal proportional to the torque and indicative of the torque's direction of action. If a simple spring were used to counteract the torque, then the deflection of the spring in inches or degrees would be directly proportional to the torque, with the spring constant, K, being the proportionality factor. By virtue of an electrical tie-in such as varying the resistance in an electrical path or reluctance in a magnetic path, it is possible to produce a voltage

output proportional to spring deflection. Hence the voltage out may be made proportional to torque in as expressed by the formula

$T = K_s K_p E_t$ where:

T = torque output of rate gyro (in lb.)

K_s = spring constant (in lb. per degree or inch)

K_p = pick-off constant (inches or degrees per volt)

E_t = voltage output of torque pick-off (volts)

As indicated previously, the rate gyro and torque pick-off performance may best be lumped in one transfer function:

$$K_{gp} G_{gp} = \frac{H}{K_s K_p} \text{ or simply } Q \left(\frac{\text{volts}}{\text{ang. vel. in rad/sec.}} \right)$$

It is to be noted that tests of a rate gyro-torque pick-off combination indicate phase shift to be negligible up to 2 cps. *

6. The Mount

The mount represents a considerable problem in analysis requiring various techniques of mechanical equivalence and attempts to lump distributed inertias and elastances.

Before considering the actual mechanics of analyzing the system constants in a mount, it is well to consider the gun's function as a resonant element in the loop. The output of the rate loop no longer is at the motor drive shaft. The output is found at the gun itself where the rate gyro mounted on the gun detects this resultant velocity. Primarily the gun represents an inertia and an elastance. The friction present is so slight as to leave the system greatly underdamped. The assumption is made that this friction may be neglected in analysis with no sizable error.

* "A Study of An Electro-Magnetic Instrument For Measuring Angular Rates in Space" by L. E. Troutman

On this basis, the transfer function of the mount may be represented as:

$$K_m G_m = \frac{K_r}{K_r + J_r S^2} = \frac{1}{1 + T_n^2 S^2}$$

K_r = equivalent gun elastance, in lb/rad

J_r = equivalent gun inertia, in lb sec^2

T_n = undamped natural period sec

$\omega_n = \frac{1}{T_n}$ = undamped natural angular velocity (rad/sec)

The inverse La Place transform of this transfer function is a sine wave of amplitude ω_n and period T_n. $(\mathcal{L}^{-1}\left[K_m G_m\right] = \omega_n \sin \omega_n t)$. Thus the mount's action is to attempt to impress its natural frequency on all forcing functions.* The effect of this action becomes increasingly and disproportionately noticeable as the natural frequency of the mount is approached. Conversely, if the mount is designed so that its resonant frequency lies well beyond the operating range of the servo loop (at least four times the highest loop frequency expected), then its effect may be disregarded. For cases where the resonant frequency of the mount lies outside the normal range of frequencies of the servo, but not so far out as to permit disregarding the gun altogether, its effect may be approximated by reference to a standard series of dimensionless curves of gain $\left(\frac{\text{output}}{\text{input}}\right)$ versus frequency ratio $\left(\frac{\text{forcing frequency}}{\text{resonant frequency}}\right)$ and phase shift versus frequency ratio which may be found in any engineers' handbook or text on vibration.** The curves form a family with the damping ratio $\left(\frac{\text{actual damping}}{\text{critical damping}}\right)$ as the dimension-

*Mechanical Vibrations, Second Edition, 1940, McGraw-Hill Book Co., Inc. New York - J. P. Den Hartog, Chapter 2, Sections 14 and 15.

** Mechanical Fibrations, Second Edition, 1940, McGraw-Hill Book Co., Inc. New York - J. P. Den Hartog, Pg. 64, Fig. 42a, 42b.

less parameter. Limited experience indicates that the curve for damping ratio of .1 is fairly representative of measured results.

When conducting tests on power drives for various types of guns in the laboratory, it is desirable that some appropriate substitute be used for the mount. This saves space and time of installation, and in most cases, allows preliminary testing of mounts which are themselves in the design process.

The problem is to compute the inertia and elastance of the mount with requisite gearing and shafting as viewed from the output of the power drive. The total reflected inertia is to be duplicated by a solid wheel which is connected to the power motor by means of a shaft, the elastance of which is equivalent to the computed elastance of the mount gearing and shafting when considered at motor speed. From the equivalent inertia and elastance the undamped natural frequency may be determined.

Two methods will be presented for the computation of this equivalent inertia and elastance. In the first case, it is assumed that the system external to the power motor is, in effect, a torsion pendulum and that the connections inside each gear box are torsionally rigid. The inertia of the shafting and the gears is assumed to be negligible. The second method includes the power motor rotor as a resonant element in the vibrating system. This method is derived from the technique used in power engineering to determine resonant frequencies of prime movers, shafts, and loads considered as single systems. Inertia of gearing is included but rigidity of gears is still assumed.

Regardless of which method of computation is used, it is essential

that a complete study of the shafting be conducted. Each shaft travels at a speed dependent upon the gear ratio at its driven end. Also, each shaft may have different diameters along its length due to couplings, gear hubs, or other design features. Therefore, it is necessary to replace each existing shaft by an equivalent shaft of uniform diameter and also to refer all shafts to the same speed level for determination of overall elastance.

The length of an equivalent shaft may be computed by choosing a diameter, converting each section of shaft to this same diameter, and adding the equivalent lengths. The individual equivalent lengths are computed by the following formula:

$$L_o = \frac{L D_o^4}{D^4} \qquad (3\text{-}1)$$

where L_o and D_o are equivalent length and diameter and L and D are existing length and diameter.

A simple illustration follows.

Assume a shaft has the form indicated in Figure 3-6.

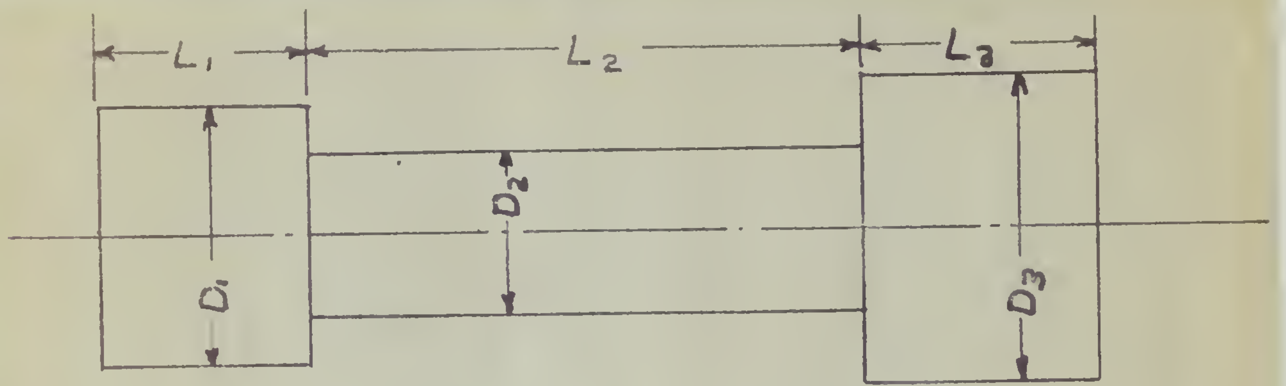


FIGURE 3-6

This shaft may be replaced by an equivalent one of uniform diameter, D_o.

The equivalent lengths of the three sections making up the shaft are computed by formula (3-1).

$$L_{01} = L_1 \frac{D_0^{\,4}}{D_1^{\,4}}$$

$$L_{02} = L_2 \frac{D_0^{\,4}}{D_2^{\,4}}$$

$$L_{03} = L_3 \frac{D_0^{\,4}}{D_3^{\,4}}$$

The length of the equivalent shaft is equal to $L_{01} + L_{02} + L_{03}$. Therefore the shaft shown in Figure 3-6 may be replaced by one of uniform diameter as indicated in Figure 3-7.

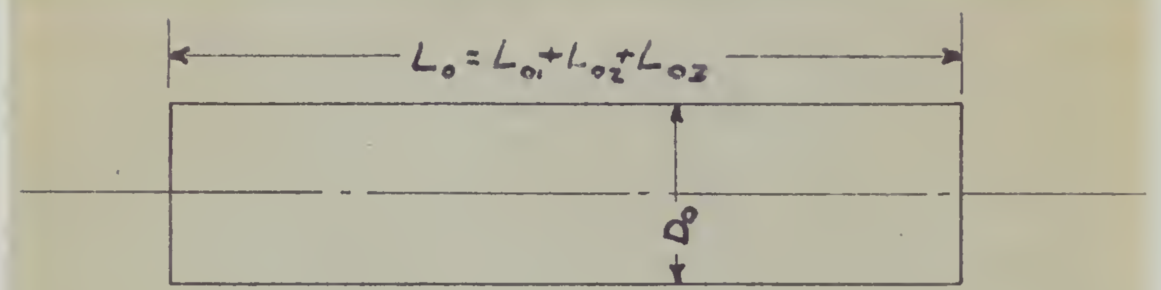


FIGURE 3-7

The elastance, designated by K, i.e., the torque in inch pounds necessary to produce an angle of twist equal to one radian, may be computed for each shaft:

$$K^1 = \frac{GI_p}{L} \qquad (3\text{-}2)$$

where G = modulus of elasticity in shear in lbs/in.^{2} (12×10^6 lb/in^2) for steel

I_p = polar moment of inertia of the circular cross-section of the shaft in (inch)^4

L = length in inches

For a circular shaft

$$I_p = \frac{\text{area x ra}}{2} = \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

Substituting this value for I_p in formula (3-2):

$$K^1 = \frac{\pi d^4 G}{32L}$$

$$K = \frac{\pi}{180} K^1 \qquad (3\text{-}3)$$

K expresses the angle of twist in degrees as compared to K^1 which expresses it in radians.

Therefore $$K = \frac{\pi^2 d^4 G}{180 \times 32L} \qquad (3\text{-}4)$$

Substituting the value for G in (3-4) and simplifying gives:

$$K = \frac{2.06 \times 10^4 d^4}{L} \quad \frac{\text{in lb}}{\text{deg}} \qquad (3\text{-}5)$$

To combine shaft elastances at various speed levels, it is necessary to select a common speed to which all shafts may be referred. This is motor speed if the laboratory substitute is to be direct drive. Shaft elastances are then combined by the formula:

$$\frac{1}{K_t} = \frac{1}{K_1} + \frac{1}{K_2\frac{(N_1)^2}{(N_2)^2}} + \frac{1}{K_3\frac{(N_1)^2}{(N_3)^2}} + \ldots. \qquad (3\text{-}6)$$

where K_t is the equivalent elastance of the system; K_1, K_2, K_3 etc. represent the elastance of individual shafts; and N_1, N_2, N_3, etc. represent the speeds of shafts 1, 2, 3. ....

Having computed K_t, a shaft having an equivalent elastance can be designed using formula (3-5) and selecting a diameter such that the length as computed will be within practical limits for the laboratory installation of this simulated gun.

The inertia, J_1, of the gun or mount is usually specified by the Bureau of Ordnance. The inertia reflected to the drive motor end is equal to the inertia of the gun divided by the square of the gear ratio.

As $J = \frac{1}{2} Mr^2$ for a solid cylinder, the mass of an equivalent wheel may be computed by assuming an appropriate value for r, the radius.

Having determined K, the elastance, and J, the inertia, for the system, ω_n, the undamped natural angular velocity may be computed as

$$\omega_n = \sqrt{\frac{K}{J}} \qquad (3\text{-}7)$$

The above analysis has permitted the replacement of a complicated system of masses, gearing, and shafting by a single one consisting of one shaft upon which is mounted one mass, as shown in Fig. 3-8, the gun simulator.

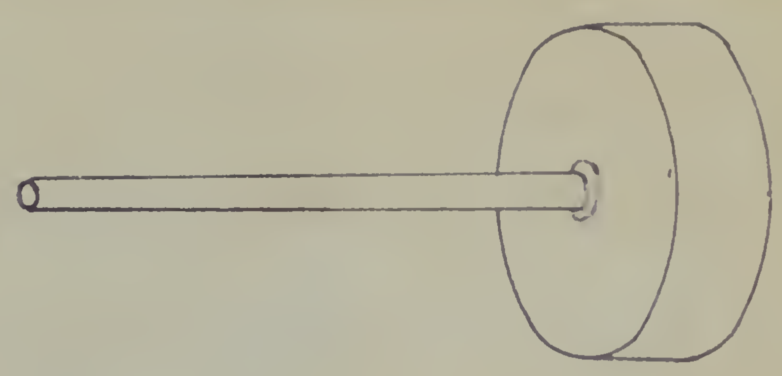


FIGURE 3-8

For the second method, the inertia of each gear is to be considered.

Here again, in order to facilitate calculations, a complex system of gearing and shafting is replaced by a simpler system consisting of a series of concentrated masses connected by sections of weightless shafting which retain as closely as possible the elastic characteristics of the original arrangement.

The mass of shafts in the gear system is usually negligible and as a general rule may be neglected if the product of the length of the shaft in feet multiplied by the frequency in vibrations per sec. does not exceed 1,000. *

As in method 1, each shaft must be reduced to a common diameter throughout its length, as indicated by formula (3-1).

The torsional vibration characteristics of the given system are then replaced by a dynamically equivalent system in which all shafts and masses rotate with the same angular velocity. To illustrate, assume gears as indicated in Figure (3-9).

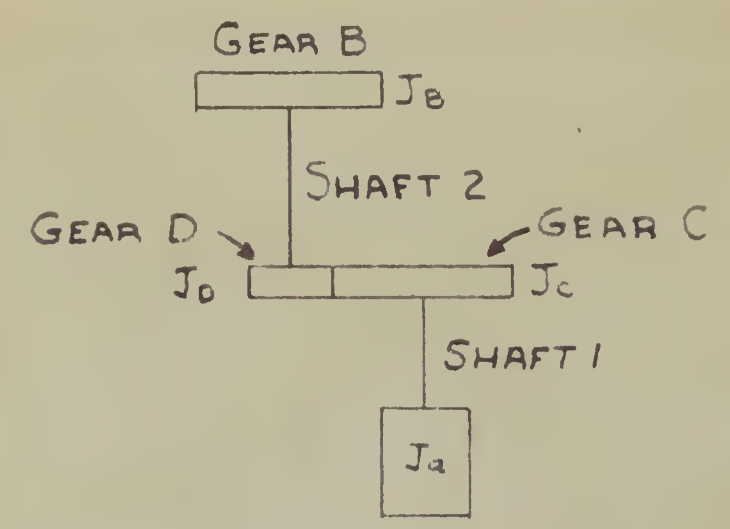


FIGURE 3-9

This system may be replaced by the one shown in Figure 3-10 which

*Reference: "Practical Solution of Torsional Vibration Problems," W. Ker Wilson, London, Chapman & Hall Ltd. 1935.

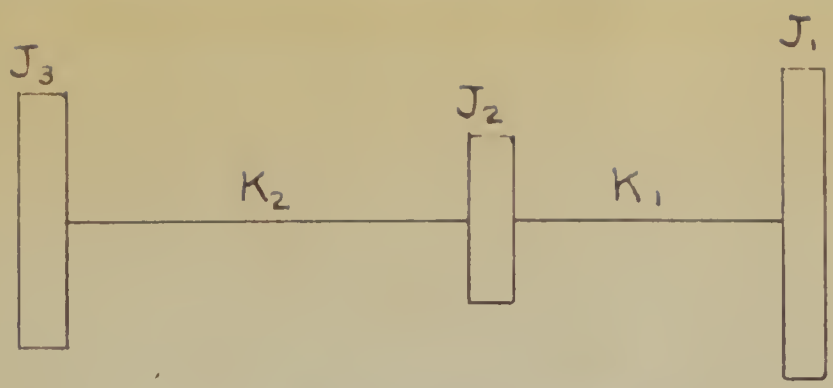


FIGURE 3-10

is essentially a single shaft with three fixed masses,

where $J_1 = J_a$

$$J_2 = J_c \quad J_d \left(\frac{N_2}{N_1}\right)^2$$

$$J_3 = J_b \left(\frac{N_2}{N_1}\right)^2$$

J in each case represents the polar moment of inertia of a mass, such as the rotor of the drive motor, or a gear; and N_1 and N_2 represent the speeds of shafts 1 and 2.

K_1 and K_2 are the equivalent elastances of shafts one and two which are computed by formula (3-5) and have been converted to the same speed by means of formula (3-6).

For a system of this type the following frequency equation exists:

$$(J_1+J_2+J_3) - \left(\frac{J_1J_2+J_1J_3}{K_1} + \frac{J_1J_3+J_2J_3}{K_2}\right)\omega_n^2 + \frac{J_1J_2J_3}{K_1K_2}\omega_n^4 = 0. \quad (3\text{-}8)\ *$$

This equation allows calculation of the two frequencies of natural vibration. In the case of a shaft with many rotating masses approximate

* "Practical Solution of Torsional Vibration Problems," W. Ker Wilson London, Chapman & Hall Ltd. 1935

numerical and graphical methods are usually applied in calculating frequencies of natural vibration.**

The theory presented above relative to computing equivalent elastances and inertias is exemplified in Appendix A by presenting a complete calculation of the natural frequencies of vibration of a typical Navy gun using both methods.

** Vol. 35, 1925, P. 109, Trans. Soc. of Naval Architects & Marine Engineers - F. M. Lewis.

CHAPTER IV

AMPLIDYNE CHARACTERISTICS

To synthesize a servo loop, the characteristics of the power drive must be known. Primarily these characteristics are the torque-speed relationship and the frequency response. The torque-speed curves are taken at various control levels. The frequency response is the magnitude and phase relationship of output to input, when the input is varied sinusoidally over a selected frequency range.

In Chapter III it was pointed out that the drive selected must be powerful enough for the task in hand and should be torque-compensated. These two criteria must be met, and the torque-speed curves and frequency response obtained, before the synthesis of the complete loop may be initiated. As stated in Chapter III, although the frequency response of an amplidyne may be estimated mathematically, it is better to obtain a response experimentally and use this graphically in all calculations.

Appendix B contains a description of equipment and method used to get the torque-speed curves and such other characteristic plots as were deemed valuable to analysis. The principal curves are presented in this Chapter (Fig. 4-1, 4-2) as typical of amplidyne performance. Supplementary curves are included in the appendix should future use of this particular amplidyne make additional data worthwhile. (Fig. B-4 through B-10 less B-7).

No calculations are shown typical of the determination that the amplidyne is powerful or fast enough for the task assigned. The problem of torque-compensation is a more interesting one. Ideally, the torque-speed curves should be horizontal straight lines, that is, speed should

be determined by the control field setting and maintained regardless of torque demand. After the work outlined in Appendix B is conducted, curves as shown in Fig. 4-1 are obtained, and the presence or lack of torque-compensation is apparent from the slope of the curves.

Two methods are available to correct this slope to nearly zero. The first, which has been mentioned in Chapter III, is to vary the compensating field effect either by physically changing the number of turns in its coil or by varying a shunting resistance across the coil and so varying the current through this winding. In the particular amplidyne selected for experimentation no such change could be made without laborious machine work. Hence the second method of torque-compensation, positive feedback to the control field, was employed. Positive feedback makes use of the torque effects on the power drive to initiate corrective action in the control circuit which action neutralizes the torque effect on speed. As an example, a brief summary of the method described in detail in Appendix C is presented.

An increase in torque always produces an increase in motor current (Fig. B-8). If a resistance is placed in series with the motor armature, then the voltage drop across this resistance is proportional to the motor current and thus to the load torque. If this voltage, through a suitable circuit, is applied to the control grids of the control amplifier output stage, then the increase in voltage will cause an increase in current output. This current output controls the amplidyne and power motor output since it is the excitation of the control field. Therefore a torque increase increases the control field and so provides more power out, allowing the speed to remain constant. By suitable adjustment of the

parameters almost complete compensation may be obtained. as indicated in Figure 4-3.

As stated in Chapter II, positive feedback is inherently unstable. Therefore, the amplification of direct axis current changes induced by varying torques must not be too great or an oscillatory system will result. This is in line with the stipulation that the torque-compensated motor must not have a resonant peak in its frequency response at low frequencies.

Thus a compromise in design is forced. Referring to Chapter II, the statement was made that torque-compensation must be complete to avoid stalling at very low input speeds when the torque is suddenly increased. Experimentation and mathematical considerations dictate that the amplification in the torque-compensating feedback loop must be limited to avoid instability. The final compensating system must satisfy both demands as closely as possible. Since the overall servo loop functions to keep speed at the desired level, it is better to incline toward poorer speed regulation in exchange for better frequency response.

An advantage of torque-compensation is that it insures that a frequency response obtained at no load will be representative of the frequency response at any other steady or variable load. So long as the torque-compensation is adequate, the requisite condition exists and the frequency response may be taken and used for further synthesis.

The frequency response of the power drive is obtained by recording simultaneously the sinusoidal control input and speed output as the frequency is varied from zero to the previously determined maximum. The maximum is determined by servo application. For gun drives it may be

accepted as 5 c.p.s. The recording of input and output may be made permanently by Brush oscillograph or temporarily by cathode ray oscilloscope allowing time to record magnitudes and phase difference. Appendix D includes a description of the equipment used in the experimental work. Figure 4-4 is a representative frequency response curve for an amplidyne and power motor, and is the curve obtained experimentally from the test amplidyne.

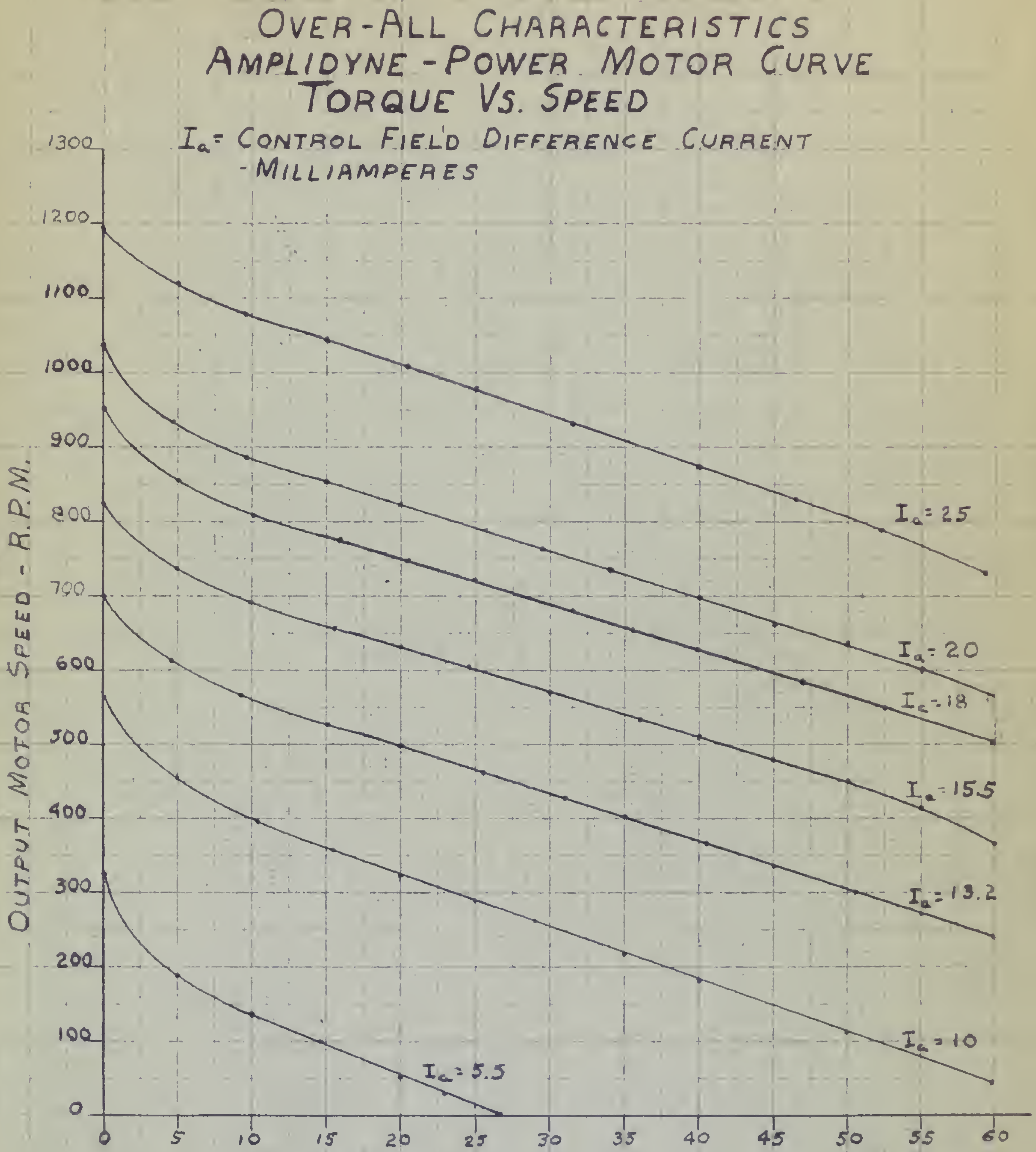
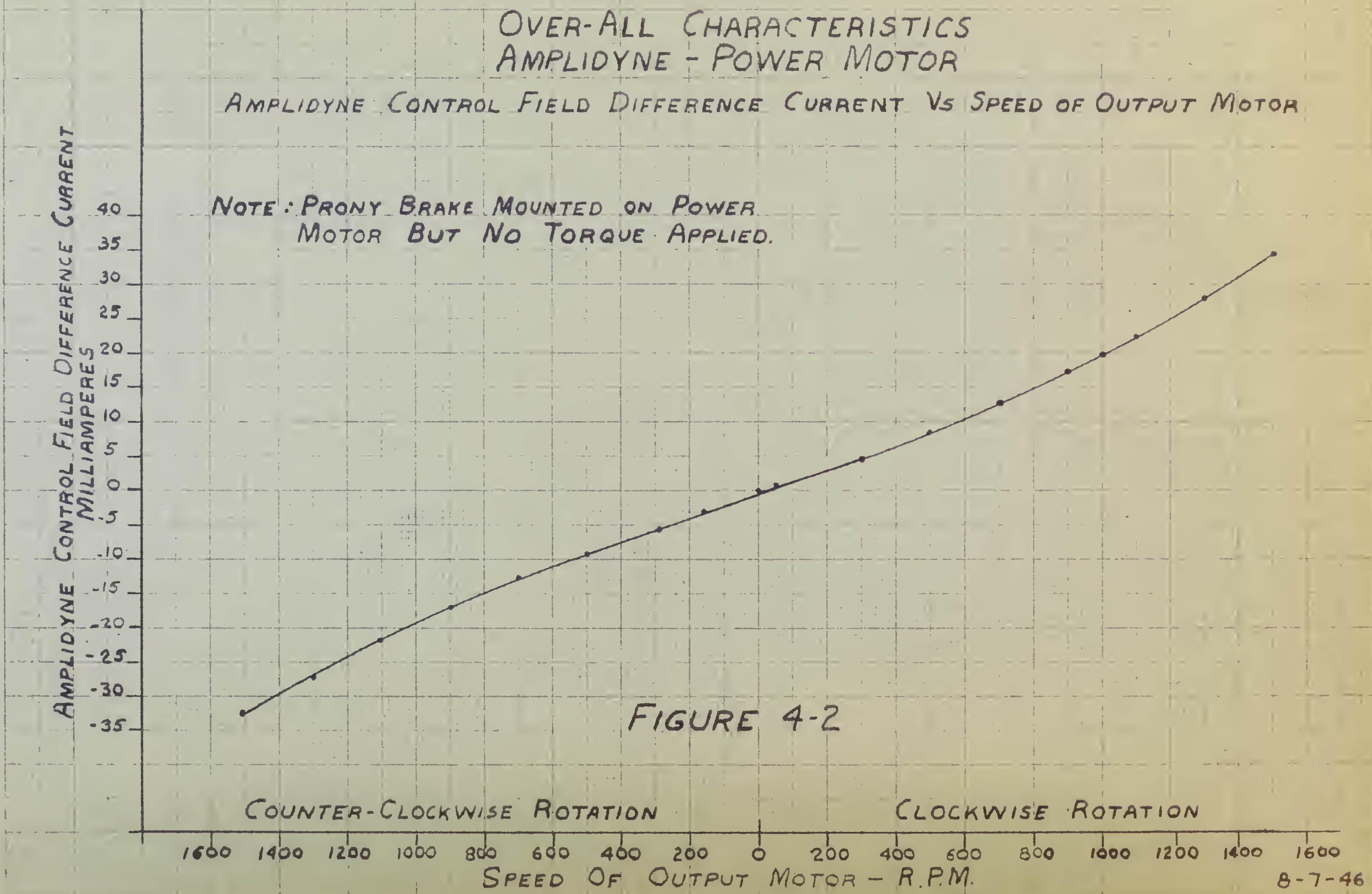


FIGURE 4-1

8-6-46



OVER-ALL CHARACTERISTICS
AMPLIDYNE - POWER MOTOR
AMPLIDYNE CONTROL FIELD DIFFERENCE CURRENT Vs SPEED OF OUTPUT MOTOR
NOTE: PRONY BRAKE MOUNTED ON POWER MOTOR BUT NO TORQUE APPLIED.
AMPLIDYNE CONTROL FIELD DIFFERENCE CURRENT
MILLIAMPERES
40
35
30
25
20
15
10
5
0
-5
-10
-15
-20
-25
-30
-35
COUNTER-CLOCKWISE ROTATION
CLOCKWISE ROTATION
1600 1400 1200 1000 800 600 400 200 0 200 400 600 800 1000 1200 1400 1600
SPEED OF OUTPUT MOTOR - R.P.M.
8-7-46

FIGURE 4-2

Torque Compensation
Torque vs Speed of Output Motor

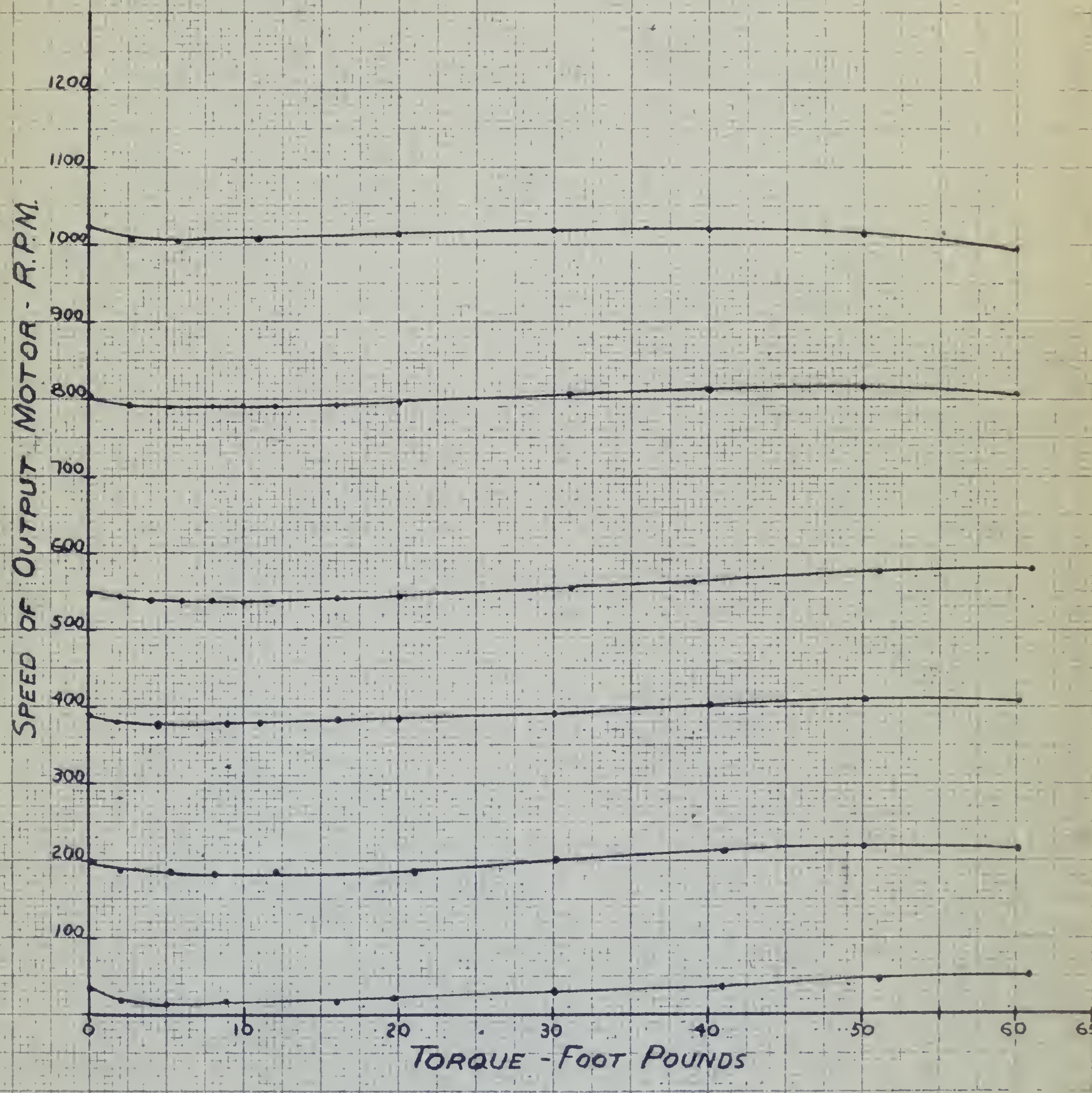
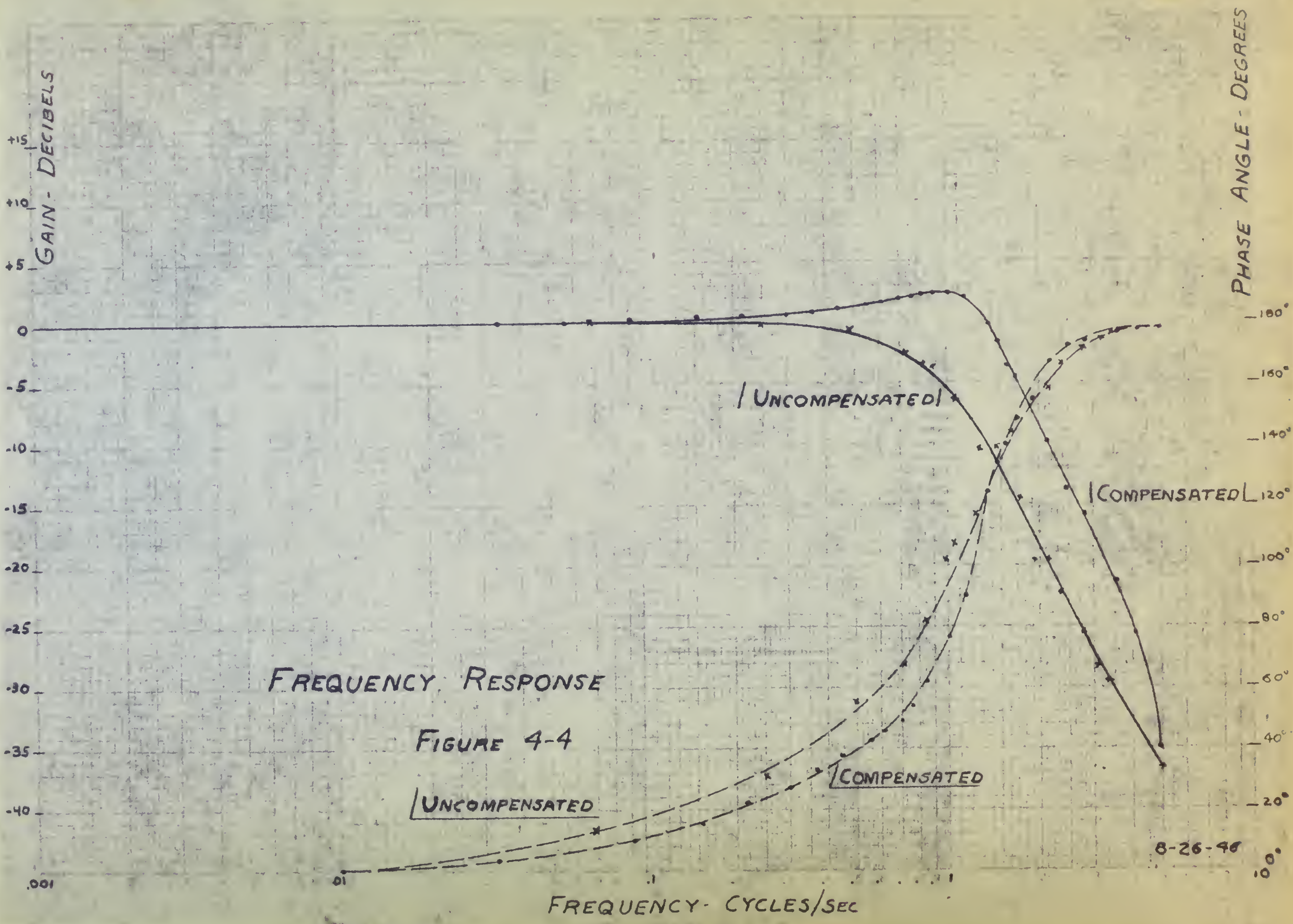


Figure 4-3

8-26-46



FREQUENCY RESPONSE

FIGURE 4-4

CHAPTER 5

SYNTHESIS OF CORRECTIVE NETWORKS

Considering the rate servo loop as presented in Chapter 3, and taking the final frequency response for the experimental amplidyne in order to make the task specific, it is desired to introduce the minimum in extra elements or networks to produce a satisfactory rate-controlled power drive for use in gun stabilization.

The position-controlled power drive in which the test amplidyne-motor was the prime mover involved considerable feedback channels in its servo loop. Since two feedback channels have already been used in securing satisfactory motor performance for use in the rate servo (See Appendices C and D) it is considered best from the standpoint of simplicity and good design technique that no feedback be used in the rate servo loop except that of the fundamental supply of output to the error-measuring device.

Since the output feedback channel is mechanical (rate gyros mounted directly on the gun) it is difficult to imagine introducing any satisfactory mechanical network in this portion of the loop. Hence investigation will be confined to the synthesis of suitable network or networks involving no electronics which may be placed in series with the error signal channel and will provide a high gain high accuracy loop over a wide band of frequencies. In Figure 5-1 the problem is shown as the synthesis of the transfer function of the box labeled K_cG_c(corrective network).

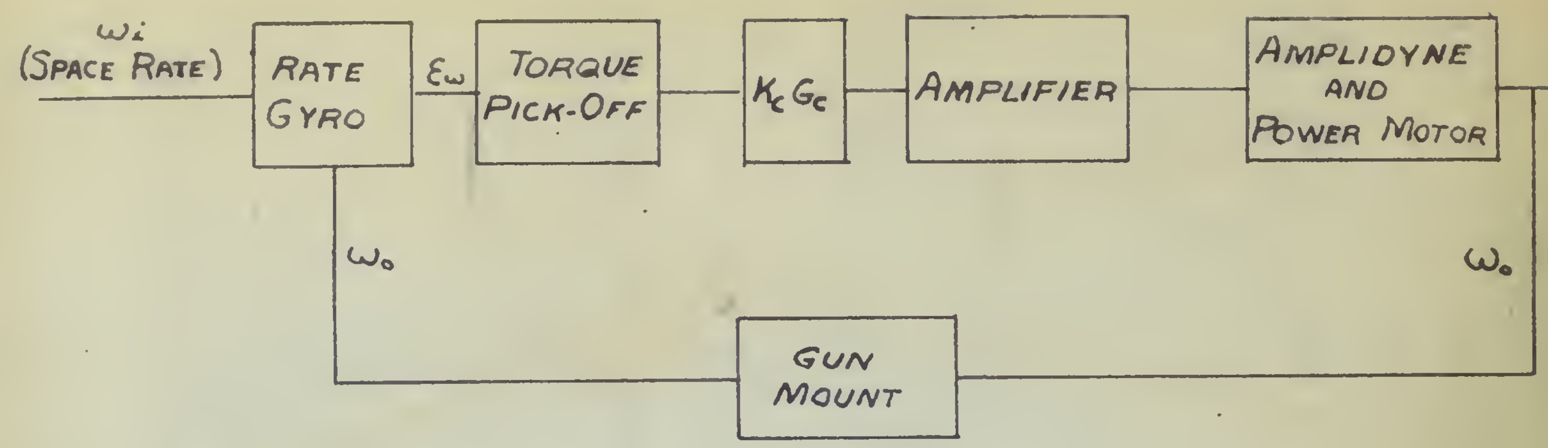


FIGURE 5-1

The transfer functions of all other elements in the loop must be considered at this point. As detailed in Chapter 3, as long as concern is with frequencies of five cycles per second or below, the transient characteristics of all other elements may be neglected. This statement is based on practical considerations. The rate gyro and torque-pickoff examined exhibited no appreciable phase shift within this frequency band. Amplifiers have time lags but this time lag may be and has been reduced to the level of thousandths of a second. The test mount as analyzed in Appendix A exhibited at the worst a resonant frequency of 41 cps which corresponds to a time lag of about .004 sec. It is realized that this last is a fortunate selection since all Navy guns are not so stiff. Nonetheless, all these transfer functions have been considered to be unity. It is a simple matter to introduce a different value should a specific application demand it. No attempt will be made to evaluate the K's individually (K is the symbol for static or steady state gain) for each element. Instead the maximum

gain which can be enclosed in the loop for desired performance will be determined. This total value may then be apportioned as is best throughout the system. Gain is most cheaply bought in the amplifier.

Before actual synthesis is begun some specifications must be formulated. Assume the worst condition of ship roll to be met is a sine wave of 20° amplitude, 9 second period. Further assume that a velocity error of 2 mils per second is tolerable. (1 mil equals one thousandth of a radian). Then the ratio of output to input is $\frac{37}{39}$ or about .95. This demands a static gain of at least 20. Since tracking inputs may reach a frequency of 3 or 4 cps, it is desired that no resonant peaks occur within or near this range. Hence the ratio of output to input should be nearly unity from 0 to about 5 cps. and phase shift negligible (less than 10°). A peak of 1.1 (about +1 db) in this ratio will be tolerable. The greatest gain possible will be enclosed while meeting the other conditions with 20 serving as a lower limit.

The method of synthesis used is the decibel-log frequency technique. Initial investigation disclosed that no ordinary combination of lead or lag networks would serve to meet the specifications. A special network suggested by Mr. William M. Pease based on the approximate quadratic response of the power drive was utilized with good results. The network devised provided a quadratic response in the numerator of its transfer function which cancelled the quadratic of the amplidyne. The denominator of this network's transfer function was also a quadratic which provided a similar frequency response with 90° phase shift occurring at a higher frequency. Hence, in effect, this network permitted moving the

resonant peak and cutoff of the amplidyne-motor to any desired point beyond that at which it naturally occurred. In conjunction with this, two lag networks providing rapid attenuation of the combined frequency response beyond .1 cps and a lead network to improve phase shift in the vicinity of 3 to 6 cps. were used. Figure 5-2 shows the log-decibel plot of initial amplidyne-motor frequency response($|A|$ is magnitude and $\angle A$ is phase, the resultant frequency response ($|R|$ and $\angle R$) with the corrective network effects included, and the ratio of output to input in the closed loop ($\left|\frac{\omega_o}{\omega_i}\right|$ and $\angle\frac{\omega_o}{\omega_i}$). Figure 5-3 presents the separate frequency response of each of the networks, "B" symbolizing one integral network, "C" the other, "Q" indicating the network whose transfer function has quadratics in both numerator and denominator, a form of lead network, and "L" representing the simple lead network.

These networks will be described in detail so that their experimental installation may be readily achieved. Both networks B and C are of the form shown in Figure 5-4.

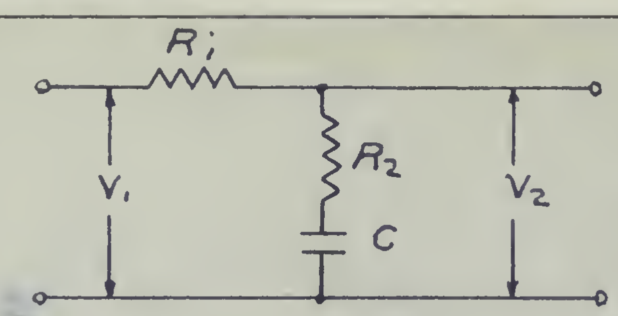


FIGURE 5-4

The transfer function of this network is:

$$\frac{V_2}{V_1} = \frac{R_2Cs+1}{(R_1+R_2)Cs+1}$$

or $$= \alpha\frac{Tis+1}{Tis+1} \quad \text{where} \quad Ti = (R_1+R_2)C$$

$$\alpha = \frac{R_2}{R_1+R_2}$$

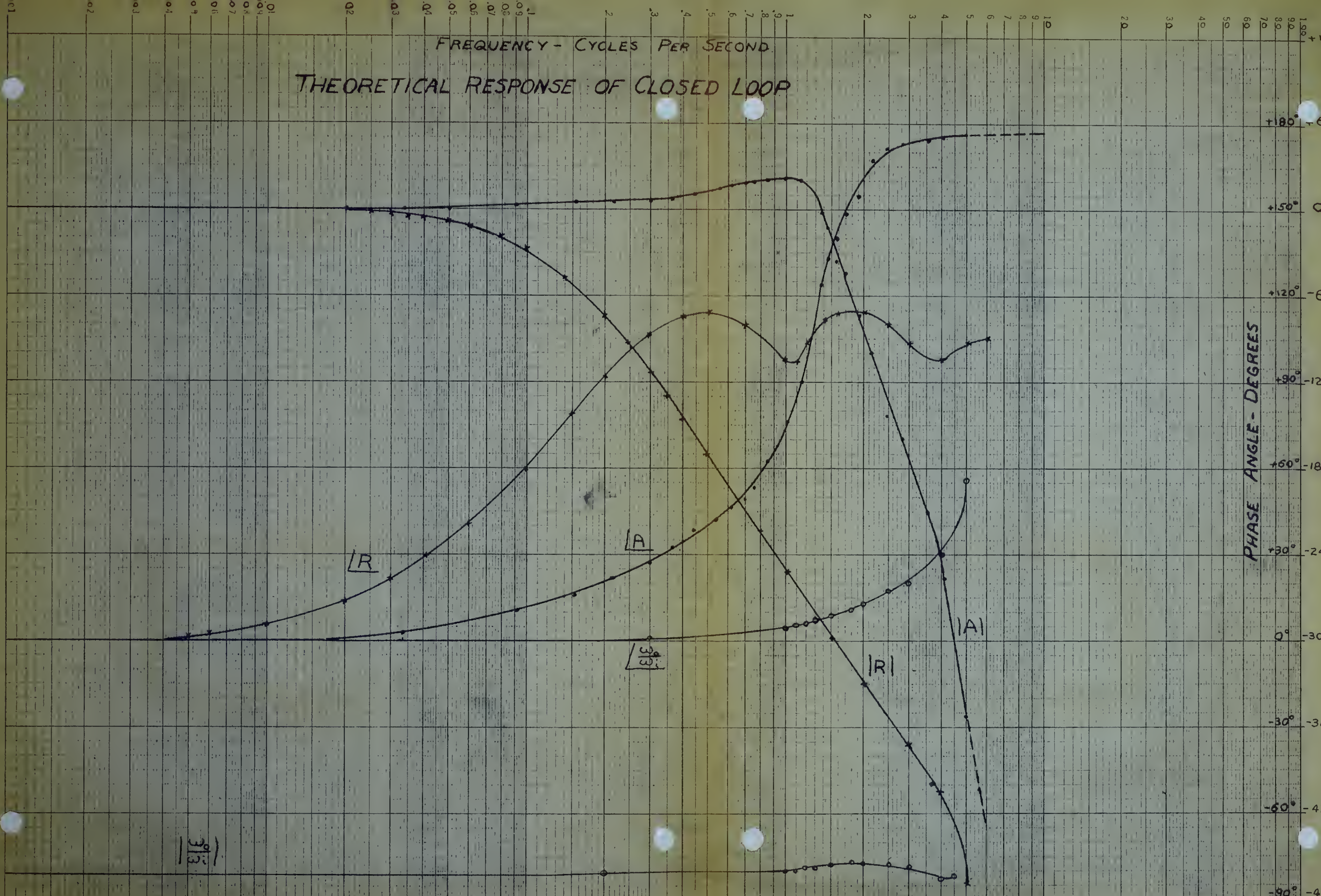
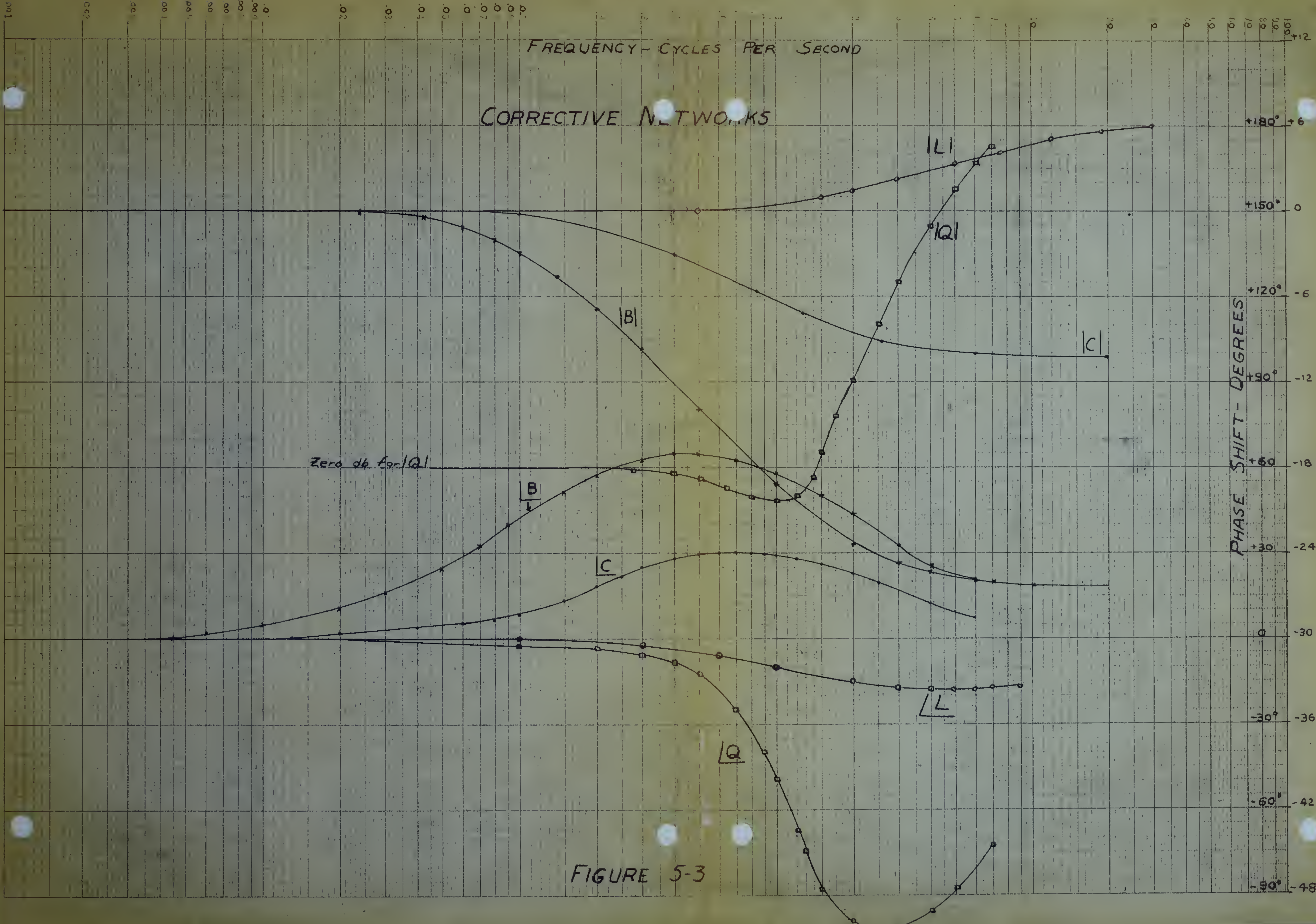

FREQUENCY - CYCLES PER SECOND
THEORETICAL RESPONSE OF CLOSED LOOP
PHASE ANGLE - DEGREES
GAIN - DECIBELS
|A|
|R|
+180°
+150°
+120°
+90°
+60°
+30°
0°
-30°
-60°
-90°
+12
+6
0
-6
-12
-18
-24
-30
-36
-42
-48

FIGURE 5-2



FREQUENCY - CYCLES PER SECOND
CORRECTIVE NETWORKS
|L|
|Q|
|B|
|C|
Zero db for |Q|
/B
/C
/L
/Q
PHASE SHIFT - DEGREES
GAIN - DECIBELS
+180°
+150°
+120°
+90°
+60
+30
0
-30°
-60°
-90°
+12
+6
0
-6
-12
-18
-24
-30
-36
-42
-48

FIGURE 5-3

In network B, Ti was selected as 1.592 seconds and ζ as .5. In network C, Ti was selected as .3975 seconds and ζ as .308. A good feature of these networks is that they have a static gain factor of unity.

Network D comprising the two quadratic factors is shown in Figure 5-5.

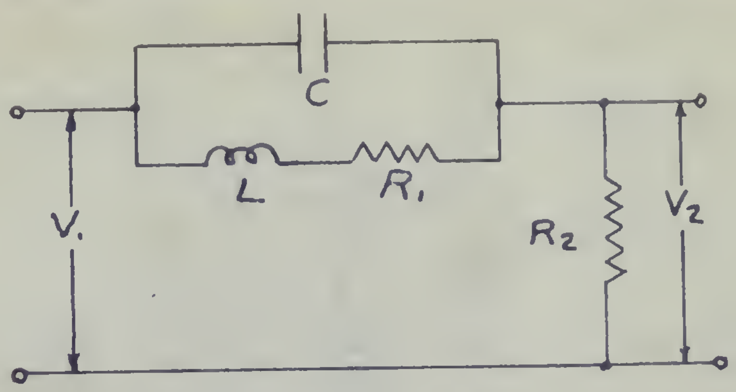


FIGURE 5-5

The transfer function is:

$$\frac{V_2}{V_1} = \frac{R_2 LCs^2 + R_2R_1Cs + R_2}{R_2CLs^2 + (R_2R_1C+L)s+R_1+R_2}$$

$$\text{or} \quad = \left(\frac{R_2}{R_1+R_2}\right) \frac{LCs^2+R_1Cs+1}{\frac{R_2}{R_1+R_2}CLs^2+\frac{R_2R_1C+L}{R_1+R_2}s+1}$$

Since it is the intention to have the numerator in effect cancel the transfer function of the amplidyne, the parameters of this quadratic are fixed. In order to relate the numerator and denominator, a mathematical transformation will be made.

Let the numerator be represented by: $T_n^2 s^2 + 2\zeta_n T_n s + 1$

where: $T_n = \sqrt{LC}$, $2\zeta_n T_n = R_1 C$

Let the denominator be represented by: $T_d^2 s^2 + 2\zeta_d T_d s + 1$

where: $T_d = \sqrt{\frac{R_2}{R_1+R_2} CL}$, $2\zeta_d T_d = \frac{R_2}{R_1+R_2}\left(R_1 C + \frac{L}{R_2}\right)$

Further, let: $\alpha = \frac{R_2}{R_1 + R_2}$. It is the intention to express the time constant and damping ratio of the denominator in terms of the numerator's parameters and the static gain factor of the network (α). This will permit complete identification of all circuit elements from knowledge of the amplidyne-motor frequency response, the selected value of the new frequency response, and a variable parameter (α) which will be adjusted to secure necessary relationships. By suitable algebra it may be shown that:

$$\alpha = \frac{R_2}{R_1+R_2} \qquad T_n = \sqrt{LC} \qquad \zeta_n = \frac{R_1}{2}\sqrt{\frac{C}{L}}$$

$$T_d = \sqrt{\alpha}\, T_n = \sqrt{\alpha LC} \qquad \zeta_D = \sqrt{\alpha}\left(\zeta_n + \frac{1-\alpha}{4\alpha\zeta_n}\right)$$

The transfer function may be written as:

$$\frac{V_2}{V_1} = \alpha \frac{T_n^2 s^2 + 2\zeta_n T_n s + 1}{\alpha T_n^2 s^2 + 2\alpha T_n \left(\zeta_n + \frac{1-\alpha}{4\alpha\zeta_n}\right) s + 1}$$

For this particular application, $T_n = .1327$, $\zeta_n = .45$, $T_d = .053$, $\zeta_d = 1$. To secure these constants it is necessary to cascade two such networks as pictured in Figure 5-5 with the intermediate quadratic (denominator of one transfer function, numerator of the other), having the constants of $T_q = .1129$, $\zeta_q = .563$. The static gain for the first network (α_1) is .724 and for the second (α_2) is .221. Hence the net gain for these two cascaded networks is .16. This imposes a greater gain requirement on other elements of the loop but should be no serious problem.

Network "L" is of the form sketched in Figure 5-6. Its sole purpose was to gain improved phase shift in the region from 1.5 to 6 cps. The effect on amplitude was not excessive in view of the great attenuation introduced by the amplidyne power drive beyond 4 cps.

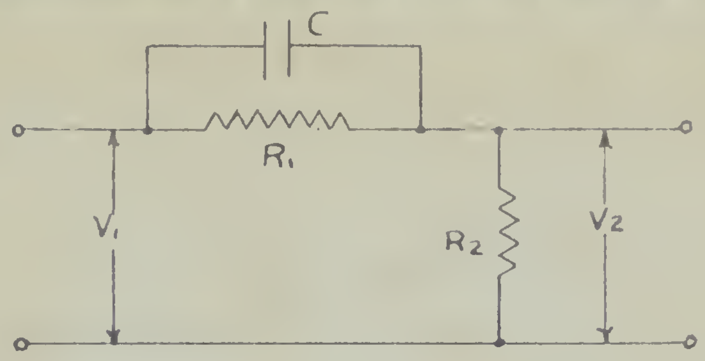


FIGURE 5-6

The transfer function is:

$$\frac{V_2}{V_1} = \frac{R_2R_1Cs + R_2}{R_2R_1Cs + R_2 + R_1}$$

$$\text{or} \quad = \frac{R_2}{R_1+R_2} \times \frac{1 + R_1Cs}{1 + \frac{R_2}{R_1+R_2} R_1Cs}$$

Let $\alpha = \frac{R_2}{R_1+R_2}$ and $T_d = R_1C$

Then: $$\frac{V_2}{V_1} = \alpha\frac{1 + T_ds}{1 + \alpha T_ds}$$

For this particular application α was selected as .5, T_d as .053 seconds.

Upon completion of the resultant plot of $\frac{\omega_o}{\epsilon}$ it is transferred to the phase margin chart from which the $\frac{\omega_o}{\omega_i}$ performance may be determined. This response as selected indicates that there will be no variation from unity as great as $\pm$1db out to 5 cycles and that in the

steady state the ratio is about -.25 db corresponding to approximately 97%. Thus with the maximum roll specified (20° amplitude, 9 sec. period) all the roll will be removed except about 1.2 mils/sec. which is better than the specified satisfactory condition. It is believe this is below the threshold of detection of a device such as the rate gyro-torque pickoff velocity error-measuring means. To secure this overall performance a gain of 46 db (about 200) is enclosed in the loop. Phase shift exceeds the desired limit becoming 10° at 1.6 cps and 30° at 4 cps.

This completes the investigation of rate-controlled and position-controlled amplidyne power drives with a view to converting from position to rate control. The succeeding chapter will review the problem and present any facts not heretofore discussed, together with such recommendations and conclusions as the authors feel are justified from this investigation.

CHAPTER 6

CONCLUSIONS

The purpose of this thesis, in general, was an investigation of position-controlled and rate-controlled amplidyne power drives. Specifically, the aim was to evaluate the two methods of control with an eye to ascertaining the difficulty of converting from one to the other. In addition, it was desired to note advantages and disadvantages so that the better control method might be selected. Wherever application of the power drive was an important factor in making decisions, the servo loop was evaluated in the light of its efficacy as power supply for a ship-borne locally stabilized gun mount.

It is regrettable that time did not permit experimental work in testing the loop whose final synthesis is presented in Chapter 5. Reliance for its practical success will be placed on the previous good results obtained when the log-db technique was used to estimate corrective networks.

Briefly, it is observed that, for a given degree of complexity, a rate servo loop will give better performance. This statement is based on the inherently greater stability of the transfer locus of the rate servo as obtained from the corresponding position servo's transfer locus. The statement is substantiated in this particular case since the theoretical performance of the rate loop synthesized is better than that of the corresponding position loop from which it is adapted.

It is also noted that, if a speed motor is used as the power drive (speed motor implies a torque-compensated prime mover), the problem of

As a special difficulty under this same general heading, it is felt that the question of the effect of couplings deserves consideration. In the example presented in Appendix A no mention was made of a special alignment coupling used to connect the drive motor to the 600 speed shaft. The stiffness of this coupling along the drive axis is fairly high (not measured) but across the axis its stiffness is negligible. Since vibration in a complex system of gearing and shafting can be troublesome whether it occurs from a torsion pendulum effect (along the drive axis) or from vibration in any other direction (across the axis), these couplings require separate investigation.

The authors' first experience in the use of the decibel-log frequency method was gained during this thesis period. It is their wish to endorse this technique as an excellent tool, especially in connection with the synthesis of servo loops.

On the problem of conversion from position to rate control where amplidynes furnish the motive power, the work done in connection with this thesis substantiates the view that it is not a simple matter, but it is vastly easier than designing all new equipment. The major feature of such conversion from the mechanical point of view is the replacement of the position servo error-measuring means with its rate servo counterpart, including perhaps a change in exciting current from 60 to 400 cps, and the installation of the sight on the mount. From an electrical point of view, considerable change in the corrective networks and the feedback paths is involved. To take advantage of the increased gain possible in the rate loop, additional stages of amplification may be required. Considering the complexity of most position servo circuits with their

obtaining a good rate servo loop is quite simple. In the example presented in Chapter 5, it was necessary only to introduce corrective networks.

The use of feedbacks around the motor gave a twofold improvement. The first was the torque compensation. The second was a better frequency response which must be interpreted both in the light of the improved gain and phase shift, and in the light of transient response.

Other values for the parameters of the corrective networks in the feedbacks paths might have given equally satisfactory frequency response, but the best transient response had to be obtained as well. This, of course, only determined the amount of damping in the loop around the motor, but it is very important from the standpoint of quickly reducing errors to a small tolerable value, and holding them there.

It was intended that additional attention be paid to the problem of the resonant element (the mount) in the primary feedback path of the rate servo loop. Although two methods of analyzing this element were presented, no great faith in the efficacy of either method is felt. It is recommended that, whenever possible, this problem receive special attention. It may be pointed out that the methods of analysis available in the Servomechanisms Laboratory should prove excellent for getting at the core of the matter. If the frequency response curves obtained by experimental work for the same rate servo with and without a resonant element in the loop are compared, the transfer function of the resonant element in question may be obtained. A satisfactory agreement between this experimental transfer locus and one obtained by mathematical analysis would eliminate large doubts now raised by this problem.

two levels of signal control, it is reasonably safe to estimate that the resulting rate servo amplifier would be no larger even with this added gain. From an operational point of view, the sight operator should be able to adjust quickly and should be favorably stimulated since riding with the mount eliminates his greatest difficulty, namely, attempting to step smoothly and evenly around the director stand. When it is considered that by such minor steps the mount is made capable of complete local operation and stabilization, the program involving these steps must be endorsed.

APPENDIX A

COMPUTATION OF EQUIVALENT INERTIA AND ELASTANCE

FOR A TYPICAL NAVY GUN MOUNT

In this appendix a typical Navy gun mount will be used to illustrate the computation necessary to replace the complex elastance and inertia of the mount by a simple shaft and cylindrical mass. The two methods developed in Chapter III will be used.

The gear system of the mount chosen is as shown in Figure A-1:

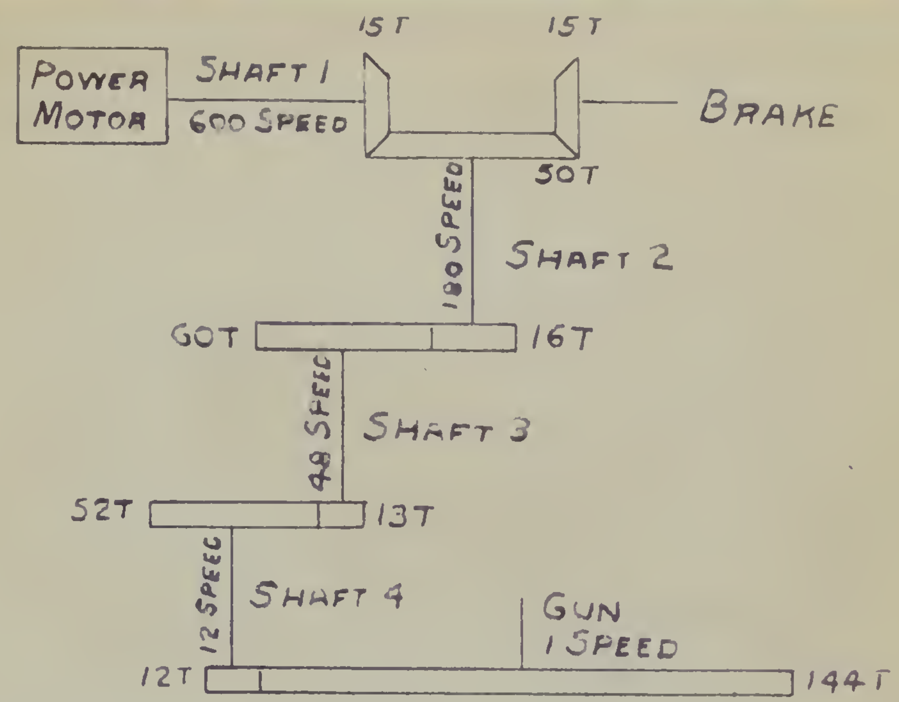


FIGURE A-1

Each shaft must be investigated to determine its diameter and length. In most cases the diameter of the individual shaft is variable so that it is first necessary to compute the length of an equivalent shaft of uniform diameter. This is done by formula 3-1, i.e.

$$L_o = \frac{L\, D_o^4}{D^4}$$

Shafts making up the gear system indicated in Figure A-1 have the forms indicated below:

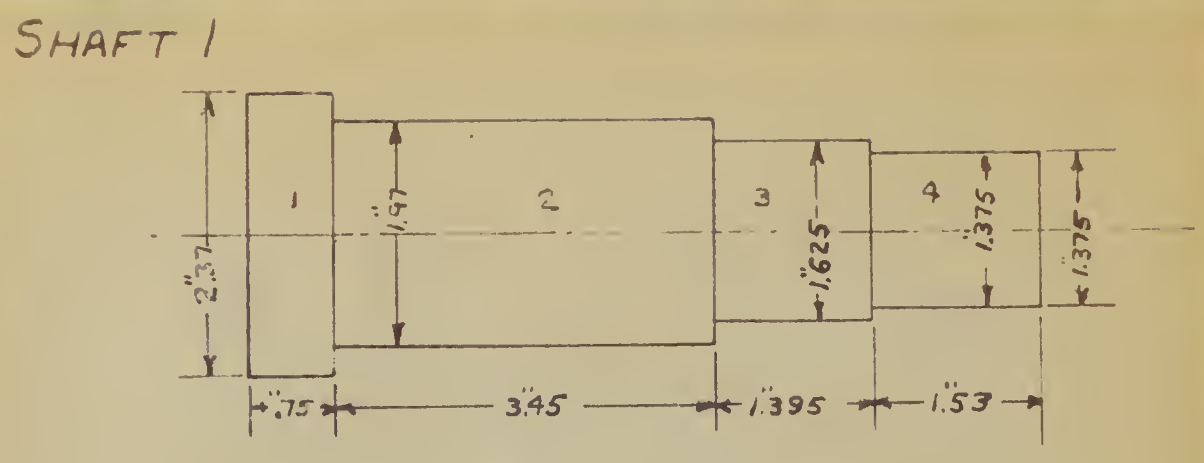


FIGURE A-2

Let $d_o = 1.97$

$$l_{o_1} = \frac{.75(1.97)^4}{(2.37)^4} = 0.36$$

$$l_{o_2} = 3.45$$

$$l_{o_3} = \frac{1.395(1.97)^4}{(1.625)^4} = 3.01$$

$$l_{o_4} = \frac{1.53(1.97)^4}{(1.375)^4} = \underline{6.44}$$

$$l_o = 13''.26 \qquad d_o = 1''.97$$

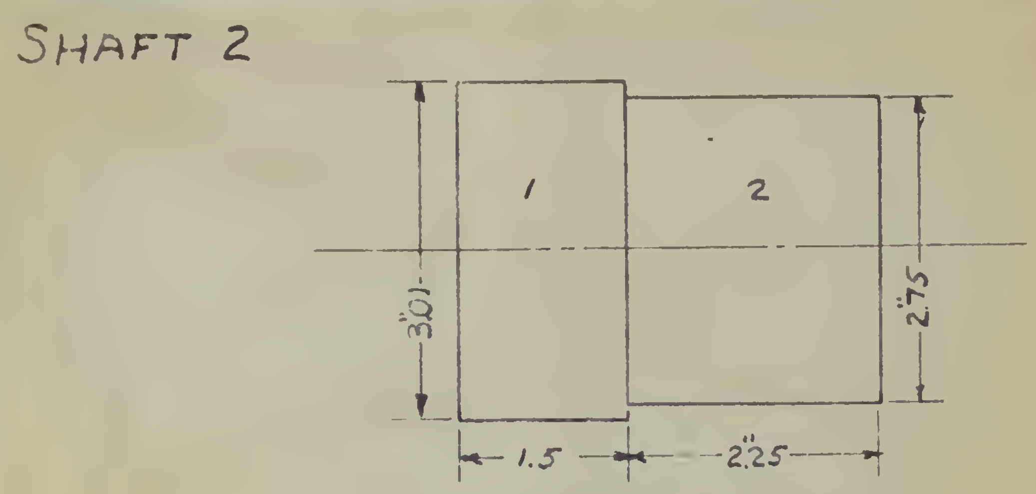


FIGURE A-3

let $d_o = 2.75$

$$l_{o_1} = 1.5\frac{(2.75)^4}{(3.01)^4} = 1.046$$

$$l_{o_2} = \qquad l_o = \frac{2.25}{3''.296} \qquad d_o = 2''.75$$

SHAFT 3

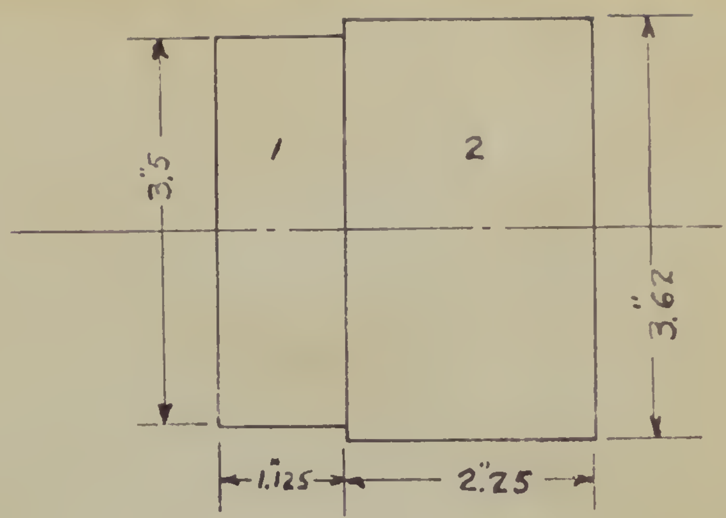


FIGURE A-4

Let $d_o = 3.62$

$$l_{o_1} = 1.125\frac{(3.62)^4}{(3.5\ )^4} = 1.28$$

$$l_{o_2} = \qquad l_o = \frac{2.25}{3''.53} \qquad d_o = 3''.62$$

SHAFT 4

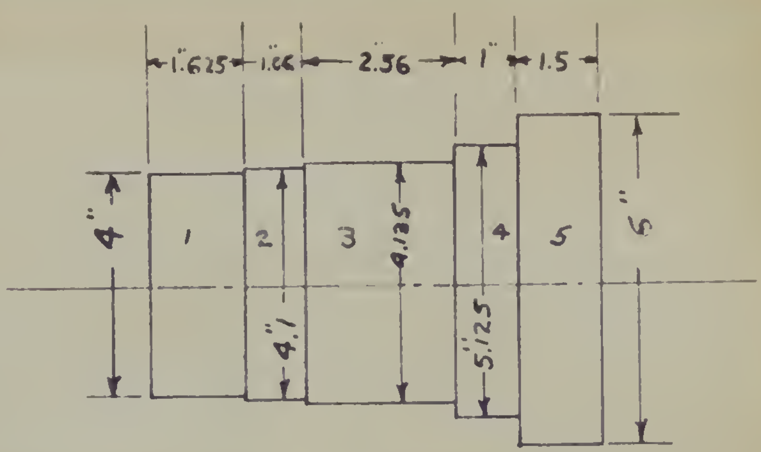


FIGURE A-5

Let $d_o = 4.135$

$$l_{o1} = 1.625 \times \left(\frac{4.135}{4}\right)^4 = 1.843$$

$$l_{o2} = 1.06\left(\frac{4.135}{4.1}\right)^4 = 1.097$$

$$l_{o3} = \qquad 2.56$$

$$l_{o4} = 1.0\left(\frac{4.135}{5.125}\right)^4 = .422$$

$$l_{o5} = 1.5\left(\frac{4.135}{6}\right)^4 = \underline{.338}$$

$$l_o = 6.260 \qquad\qquad d_o = 4.135$$

<u>METHOD I</u>

The elastance for each shaft may be computed by formula 3-5, i.e.

$$K = \frac{2.06 \times 10^4 d^4}{L}$$

$K =$ elastance in inch. lb./degree

$d =$ diameter of shaft in inches

$L =$ length of shaft in inches

1. <u>Shaft 1</u>

$$K_1 = 2.06 \times 10^4 \frac{(1.97)^4}{13.26} = 2.34 \times 10^4 \text{ in lb/degree}$$

$$\frac{1}{K_1} = .428 \times 10^{-4}$$

Speed of shaft $= 600$

2. <u>Shaft 2</u>

$$K_2 = 2.06 \times 10^4 \frac{(2.75)^4}{3.296} = 35.78 \times 10^4 \text{ in lb/degree}$$

$$1/K_2 = .028 \times 10^{-4}$$

Speed of shaft $= 180$

3. Shaft 3

$$K_3 = 2.06 \times 10^4 \times \frac{(3.62)^4}{3.53} = 100 \times 10^4 \text{ in lb/degree}$$

$$1/K_3 = .01 \times 10^{-4}$$

Speed of shaft = 48

4. Shaft 4

$$K_4 = 2.06 \times 10^4 \frac{(4.135)^4}{6.26} = 95.7 \times 10^4$$

$$1/K_4 = .014 \times 10^{-4}$$

Speed of shaft = 12

The equivalent elastance of the gun mount is computed by means of formula (3-6).

$$\frac{1}{K_t} = \frac{1}{K_1} + \frac{1}{K_2\left(\frac{N_1}{N_2}\right)^2} + \frac{1}{K_3\left(\frac{N_1}{N_3}\right)^2} + \ldots\ldots$$

Substituting the computed values of K_1, K_2, K_3, and K_4 and the speed of each shaft as indicated:

$$\frac{1}{K_t} = .428 \times 10^{-4} + .028 \times 10^{-4} \frac{(180)^2}{(600)^2} + .01 \times 10^{-4} \frac{(48)^2}{(600)^2} + .014 \times 10^{-4} \frac{(12)^2}{(600)^2}$$

$$\frac{1}{K_t} = .428 \times 10^{-4} + .00252 \times 10^{-4} + .000064 \times 10^{-4} + .0000056 \times 10^{-4}$$

$$= .4306 \times 10^{-4}$$

$$K_t = 2.32 \times 10^4 \text{ in. lb/degree}$$

$$K_t^1 = 0.1328 \times 10^7 \text{ in lb/rad.}$$

Having computed K_t, a shaft having an equivalent elastance can be designed using formula (3-5). Assume the desired diameter for such a shaft to be 2."125.

$$K = \frac{2.06 \times 10^4 d^4}{L}$$

where $K = K_t = 2.32 \times 10^4$ in lb/degree

$$2.32 \times 10^4 = \frac{2.06 \times 10^4 (2.125)^4}{L}$$

$L = 18''00$ -length of designed shaft

The inertia, J, specified for the gun mount under investigation is 350,000 lbs. ft². This inertia must be referred to the same speed level as the power motor. The inertia of the gun as viewed at the motor is $\frac{350,000}{(600)_2}$ or $\frac{35}{36}$ lb. ft.² .

It is possible to replace the inertia of the gun mount by an equivalent cylinder of appropriate mass and radius. For a cylinder:

$$J = \frac{1}{2} Mr^2$$

J = moment of inertia in lbs. ft.²

M = mass in lbs.

r = radius in feet

If r, the radius of such a wheel, is assumed equal to one-half foot then:

$$M = \frac{2J}{r^2} = \frac{2 \times \frac{35}{36}}{1/4} = \frac{70}{9} \text{ lbs.}$$

M = 7.78 lbs.

Density of steel is 0.2855 lbs/in^3 so that 7.78 lbs. is equivalent to 27.3 in^3.

As the volume of a cylinder is $\pi r^2 h$, and r is assumed to be one-half foot, the height, h, of the cylinder may be computed

$$h = \frac{27.3}{36\pi}$$

$$= 0''241$$

The undamped natural angular velocity, ω_n, may now be computed by formula (B-7), i.e.

$$\omega_n = \sqrt{\frac{K}{J}}$$

$$= \sqrt{\frac{0.1328 \times 10^7 \times 386}{140}}$$

$$\omega_n = 1910 \text{ rad/sec}$$

$$f_n = \frac{1910}{2\pi} = 304 \text{ c.p.s.}$$

As a result of the above calculation, the original complicated system of masses, gearing, and shafting composing the gun mount may be replaced by the single system indicated in Figure A-6.

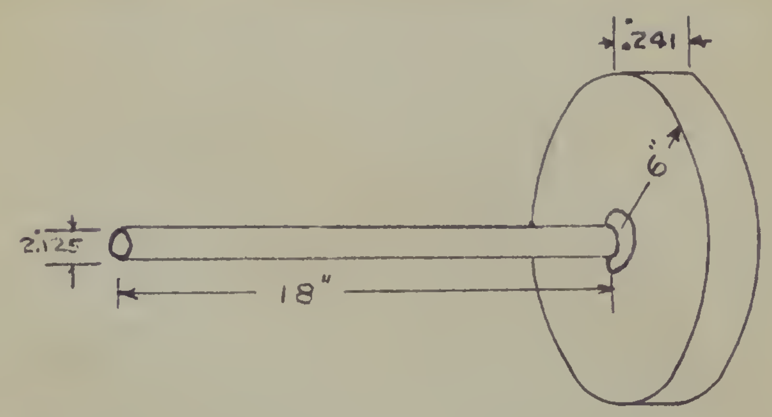


FIGURE A-6

Method 2

Figure A-7 shows the gear system of the gun mount under investigation which is identical with that depicted in Figure A-1 except that the inertia of the power motor and the gearing itself is not neglected.

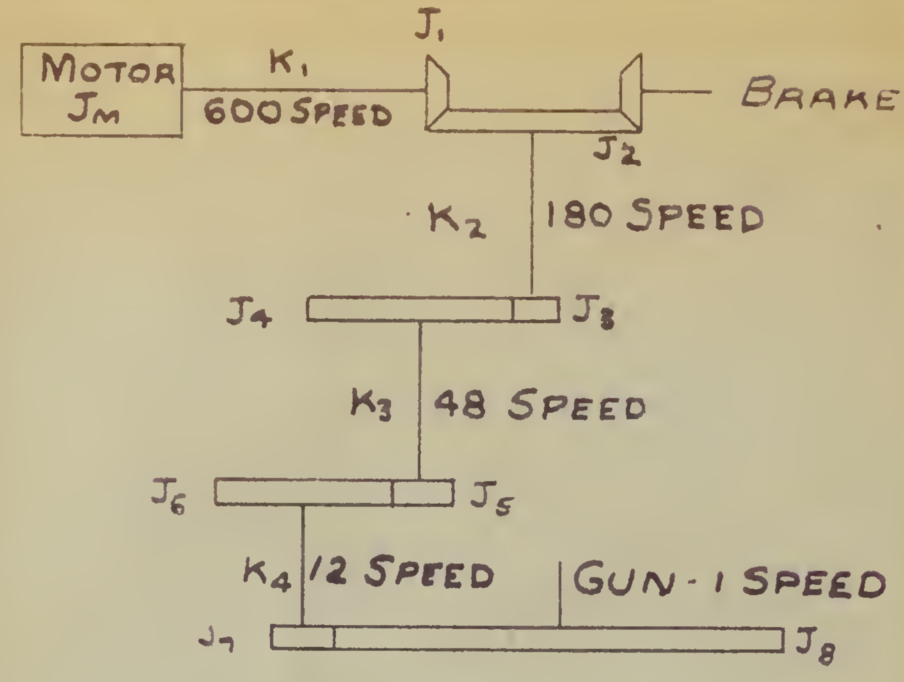


FIGURE A-7

Equivalent lengths and diameters for each shaft are computed as indicated in the first part of this appendix.

The inertia of each gear is computed from the appropriate gear diagram, using the formula:

$$J = \frac{\pi r^4}{2} da$$

where d = density in lbs/in.^{3}

a = cylinder height in inches.

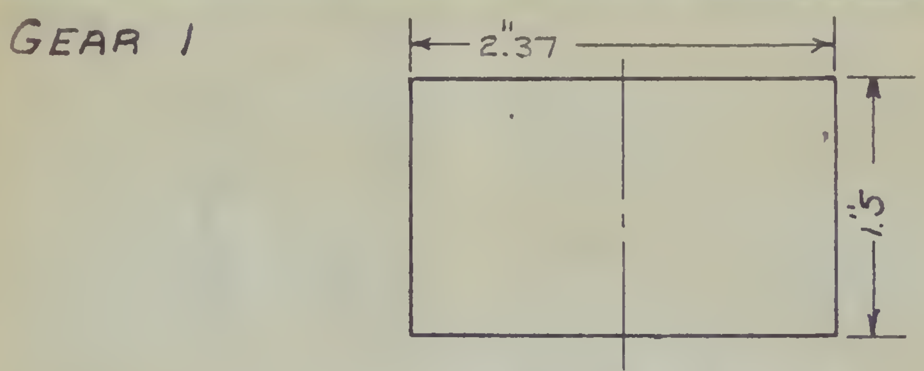


FIGURE A-8

$$J_1 = \frac{\pi}{32}(2.37)^4 \times .2855 \times 1.5 = 1.322$$

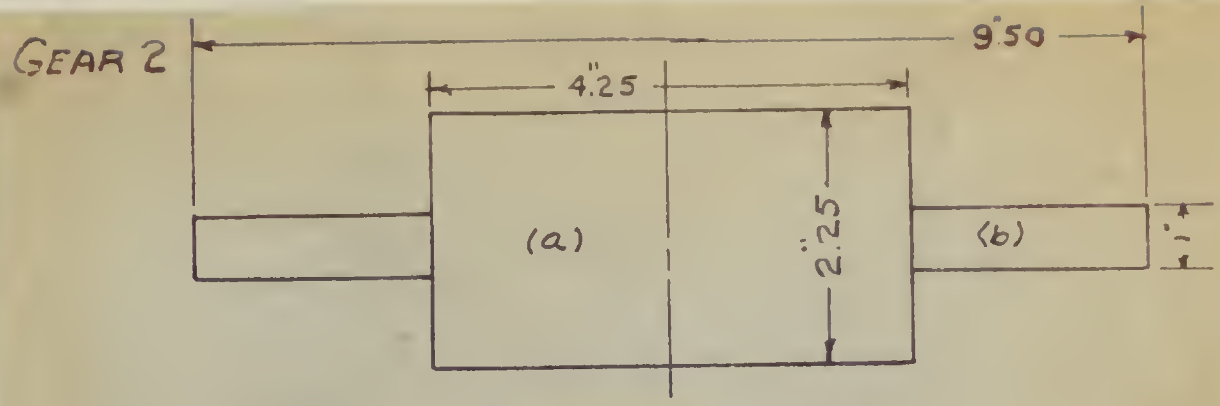


FIGURE A-9

$$J_{2a} = \frac{\pi(4.25)^4}{32} \times .2855 \times 2.25 = 20.6$$

$$J_{2b} = \frac{\pi}{2}\left[(4.75)^4 - (2.125)^4\right] .2855 \times 1 = 222$$

$$J_2 = J_{2a} + J_{2b}$$

$$J_2 = 222 + 20.6 = 242.6 \text{ lb. in}^2$$

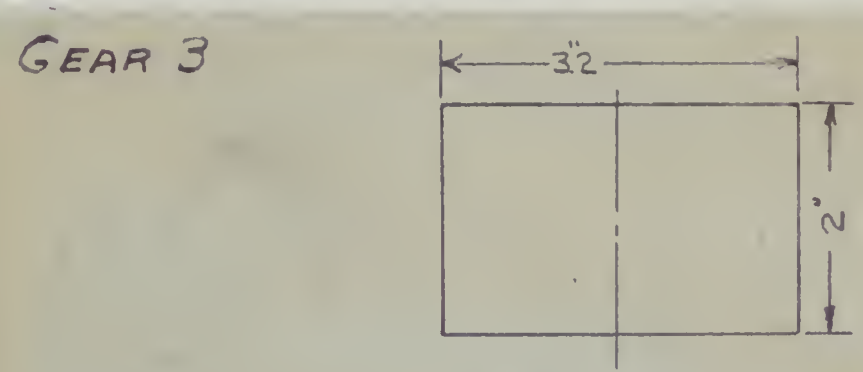


FIGURE A-10

$$J_3 = \frac{\pi(3.2)^4}{32} \times .2855 \times 2 = 5.89 \text{ lb. in}^2$$

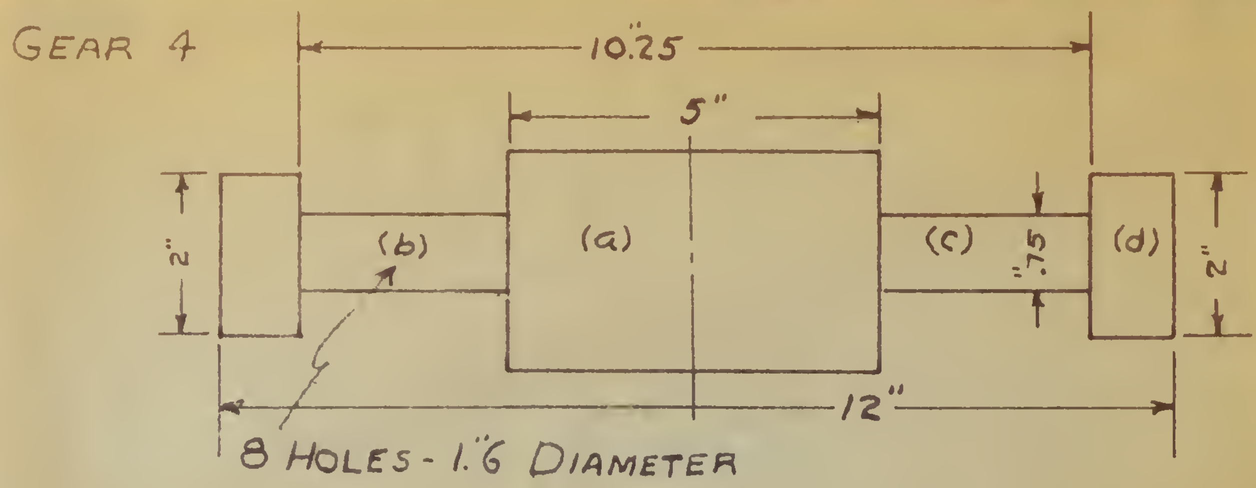


FIGURE A-11

$$J4_a = \frac{\pi}{32}(5)^4 \; .2855 \times 2.375 = 41.6$$

$$J4_b = 8\left[\frac{\pi(1.62)^4 \; .2855}{32} + \frac{\pi(1.62)^2(.2855)(4.81)^2}{4}\right](.75)$$

$$= 83$$

$$J_{4c} = \frac{\pi}{32}\left[(10.25)^4 - (5)^4\right](.2855)(.75) = 220$$

$$J_{4d} = \frac{\pi}{32}\left[(12)^4 - (10.25)^4\right] \; .2855 \times 2 = 541$$

$$J_4 = J4_a + J4_c + J4_d - J4_b$$

$$J_4 = 41.6 + 220 + 541 - 83 = 719.6 \text{ lb. in}^2$$

GEAR 5

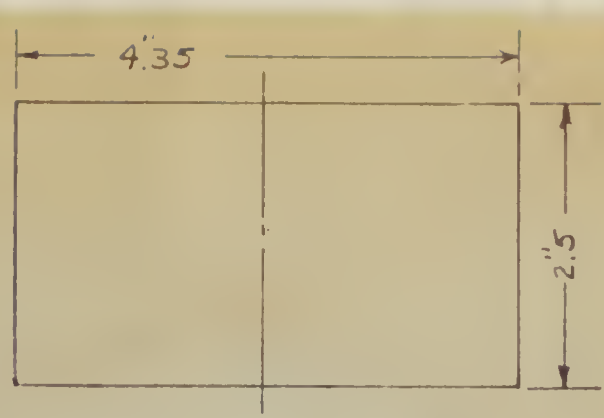


FIGURE A-12

$$J_5 = \frac{\pi}{32}(4.35)^4 \times .2885 \times 2.5 = 25.2 \text{ lbs. in}^2$$

GEAR 6

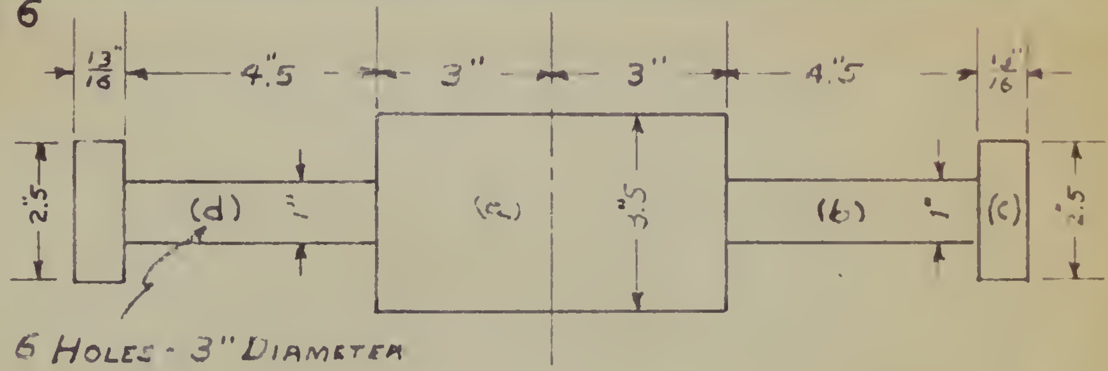


FIGURE A-13

$$J_{6a} = \frac{\pi}{2}(3)^4 \times .2855 \times 3.5 = 127.1$$

$$J_{6b} = \frac{\pi}{2}\left[(7.5)^4 - (3)^4\right] \times .2855 \times 1 = 1380$$

$$J_{6c} = \frac{\pi}{2}\left[(8.312)^4 - (7.5)^4\right] \times .2855 \times 2.5 = 1813$$

$$J_{6d} = 6\left[\frac{\pi}{32}(3)^4 \times .2855 \times 1 + .2855(1.5)^2(5.25)^2\pi\right]$$

$= 347.9$

$J_6 = J_{6a} + J_{6b} + J_{6c} - J_{6d}$

$= 127.1 + 1380 + 1813 - 347.9$

$J_6 = 3320.1 - 347.9 = 2972.2 \text{ lb. in}^2$

GEAR 7

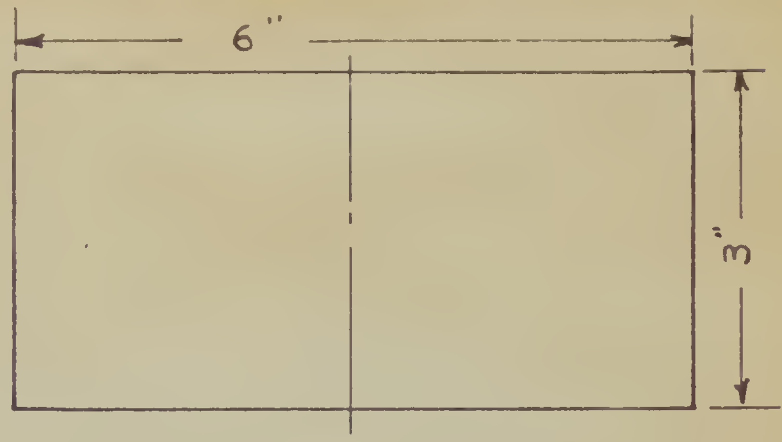


FIGURE A-14

$$J_7 = \frac{\pi (6)^4}{32} \times .2855 \times 3$$

$J_7 = 109 \text{ lb. in}^2$

Gun Mount

J_8 = Inertia of gun mount

$= 350{,}000 \text{ lb. ft.}^2$

Drive Motor Rotor

$J_m = 3.75 \text{ lb. ft.}^2$

$= 540 \text{ lb. in.}^2$

Data furnished by Bureau of Ordnance.

Referring the inertia of each gear and of the gun mount to the 600 speed (motor) level:

$J_1 = 1.322$ lbs. in^2

$J_2 = 242.6 \dfrac{(N_2)^2}{(N_1)^2}$

$= 242.6 \dfrac{(180)^2}{(600)^2} = 21.9$ lb. in^2

$J3 = 5.89\dfrac{(N_2)^2}{(N_1)^2} = 5.89 \dfrac{(180)^2}{(600)^2} = .531$ lb. in^2

$J4 = 719.6\dfrac{(N3)^2}{(N_1)^2} = 719.6\dfrac{(48)^2}{(600)^2} = 4.60$ lbs. in^2

$J_5 = 25.2\dfrac{(N3)^2}{(N_1)^2} = 25.2\dfrac{(48)^2}{(600)^2} = .1611$ lb. in^2

$J_6 = 2972.2x\dfrac{(N4)^2}{(N_1)^2} = 2972.2\dfrac{(12)^2}{(600)^2} = 1.19$ lb. in^2

$J_7 = 109\dfrac{(N4)^2}{(N_1)^2} = 109\dfrac{(12)^2}{(600)^2} = .0436$ lb. in^2

$J_8 = \dfrac{350,000}{360,000} \times 144 = 140$ lb. in^2

The resultant inertia for meshing gears is equal to the sum of the inertia of the gears in the mesh:

$J_{12} = J_1 + J_2 = 1.322 + 21.9 = 23.222$ lb. in^2

$J_{34} = J3 + J4 = .531 + 4.60 = 5.131$ lb. in^2

$J_{56} = J_5 + J_6 = .1611 + 1.19 = 1.3511$ lb. in^2

$J_{78} = J_7 + J_8 = .0436 + 140 = 140.0436$ lb. in^2

$J_m = 3.75 \times 144 = 540$ lb. in^2

The elastance, K, for each shaft as previously calculated must be referred to the 600 speed shaft:

$K_1 = 2.06x10^4 \left(\dfrac{1.97}{47.1}\right)^4 = .66x10^4$ in lb/degree

$K_2 = 35.78x10^4 \; x\dfrac{(180)^2}{(600)^2} = 3.2202x10^4$

$K3 = 100x10^4 x\dfrac{(48)^2}{(600)^2} = .64x10^4$

$K4 = 96.2x10^4 x\dfrac{(12)^2}{(600)^2} = .03848x10^4$

Using the values for the inertia and elastance computed above for the 600 speed level, the gun mount system of Figure A-7 may be replaced by the simpler form as indicated in Figure A-15.

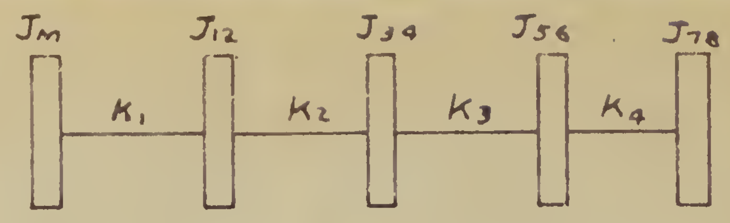


FIGURE A-15

J_{34} and J_{56} are small enough to be assumed negligible so that the five mass system shown in Figure A-15 reduces to the three-mass system of Figure A-16.

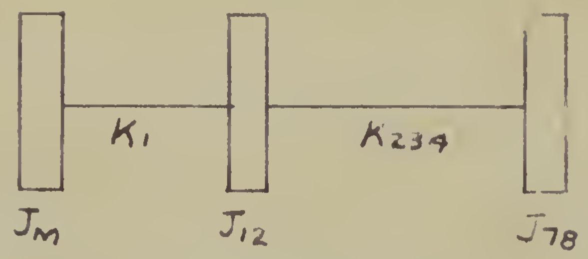


FIGURE A-16

K_{234} is the equivalent elastance of shafts 2, 3 and 4:

$$\frac{1}{K_{234}} = \frac{1}{K_2} + \frac{1}{K_3} + \frac{1}{K_4} \qquad \text{(I-3)}$$

$$= 10^{-4}\left[.31+1.56+26\right] = 10^{-4}\,(27.87)$$

$$K_{234} = .03585 \times 10^{4} \text{ in lb./degree}$$

The undamped natural angular velocity, ω_n, is computed by means of formula (3-8), i.e.

$$J_m + J_{12} + J_{78} - \omega_n^2 \left[\frac{J_m J_{12}}{K_1} + \frac{J_m J_{78}}{K_1} + \frac{J_m J_{78}}{K_{234}} + \frac{J_{12} J_{78}}{K_{234}} \right] + \omega_n^4 \left[\frac{J_m J_{12} J_{78}}{K_1 K_{234}} \right] = 0$$

Substituting computed values of inertias and elastances:

$$540 + 23.222 + 140.044 - \frac{\pi \omega_n^2}{180 \times 386} \left[\frac{540}{.66 \times 10^4}(23.22 + 140.044) + \frac{140.044(23.22 + 540)}{.03585 \times 10^4} \right]$$

$$+ \frac{\omega_n^4 \pi^2}{(180 \times 386)^2} \left[\frac{540 \times 23.22 \times 140.044}{.66 \times .03585 \times 10^8} \right] = 0$$

$$703.3 - \frac{\pi \omega_n^2}{180 \times 386} \left[\frac{540 \times 163.3}{.66 \times 10^4} + \frac{140.044 \times 563.22}{.03585 \times 10^4} \right] + \frac{\omega_n^4 \pi^2}{(180 \times 386)^2} \left[\frac{540 \times 23.2 \times 140}{.66 \times .03585 \times 10^8} \right]$$

$$= 0$$

$$.152 \times 10^{-8} \omega_n^4 - .01055 \omega_n^2 + 703.3 = 0$$

$$\omega_n^4 - 6.94 \times 10^6 \omega_n^2 + .4625 \times 10^{12} = 0$$

$$\omega_n^2 = .0673 \times 10^6 \text{ rad/sec}, \ 6.8727 \times 10^6 \text{ rad/sec}$$

$$\omega_n = 260 \text{ rad/sec}, \ 2620 \text{ rad/sec}$$

$$f_n = \frac{\omega_n}{2\pi} = 41.4 \text{ cps}, \ 418 \text{ cps}$$

APPENDIX B

AMPLIDYNE CHARACTERISTICS

Experimental work for this thesis was conducted using equipment furnished by the Bureau of Ordnance, U. S. Navy. The amplidyne, power motor, and associated components were manufactured by the General Electric Company and constitute a position-controlled gun power drive in current use by the Navy. Figure B-1 indicates such of this equipment as was used, together with the interconnections. A brief description of components follows:

1. Drive Motor and Amplidyne

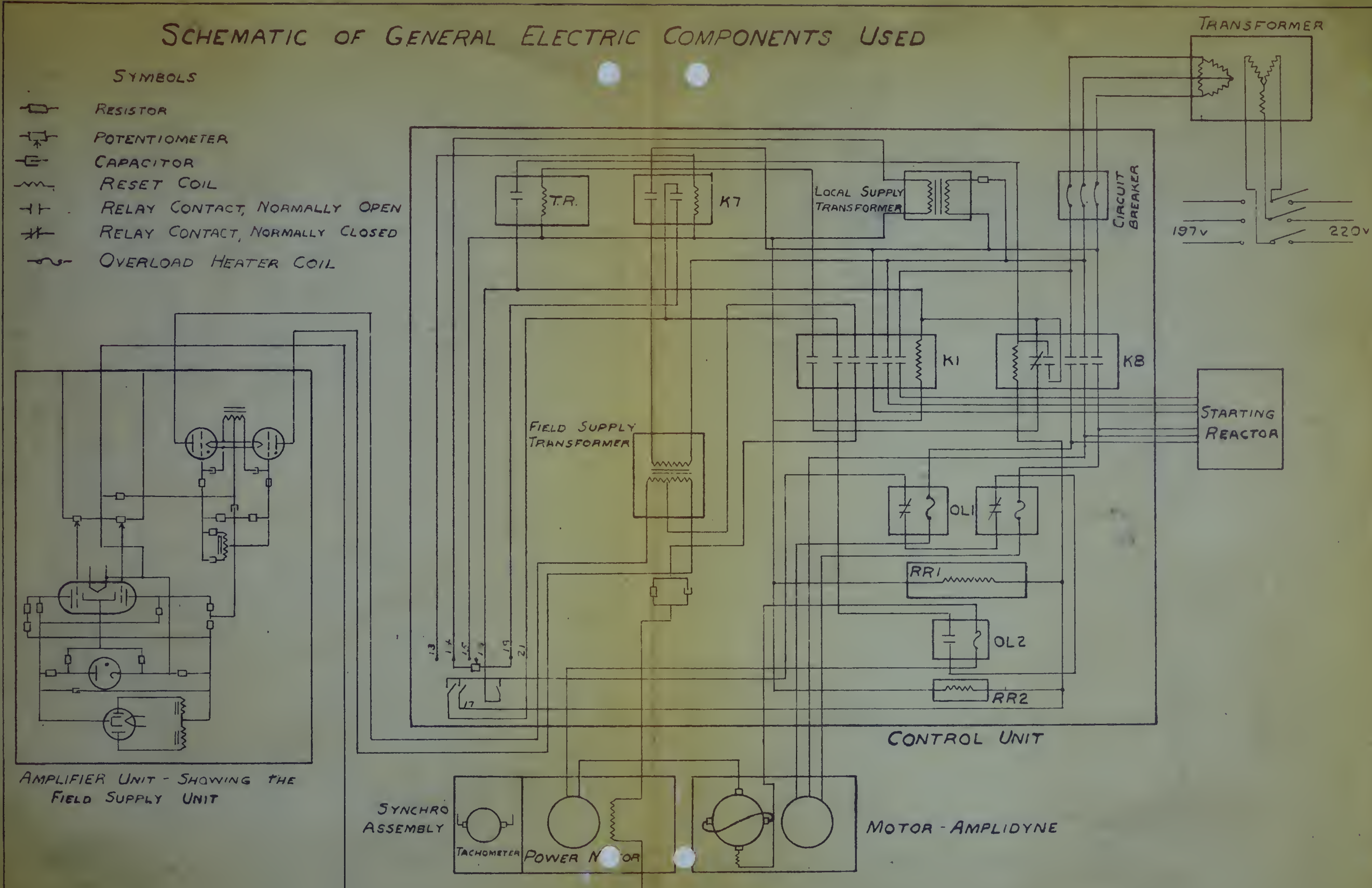
The drive motor and amplidyne comprise a single unit. The motor itself is an induction type operated from a 440 volt, 3 phase supply. The amplidyne is of general design, containing a fixed compensating field providing partial torque compensation.

2. Synchro Assembly

The synchro assembly contains two synchro control transformers and a tachometer generator. It is mounted directly on the output motor frame. The tachometer is driven by a 2 to 1 step-down from the output motor. By means of this tachometer a voltage is obtained which is directly proportional to power motor speed (See Figure B-6).

3. Power Motor

The power motor is a direct current motor of conventional design. It receives armature current directly from the amplidyne. Its field winding is energized by a constant direct current from the field supply unit in the amplifier. The direction of rotation of the motor as well as the speed is dependent on the output of the amplidyne.



SCHEMATIC OF GENERAL ELECTRIC COMPONENTS USED
SYMBOLS
RESISTOR
POTENTIOMETER
CAPACITOR
RESET COIL
RELAY CONTACT, NORMALLY OPEN
RELAY CONTACT, NORMALLY CLOSED
OVERLOAD HEATER COIL
TRANSFORMER
T.R.
K7
LOCAL SUPPLY TRANSFORMER
CIRCUIT BREAKER
197v
220v
K1
K8
STARTING REACTOR
FIELD SUPPLY TRANSFORMER
OL1
RR1
OL2
RR2
13
14
15
19
21
17
CONTROL UNIT
AMPLIFIER UNIT - SHOWING THE FIELD SUPPLY UNIT
SYNCHRO ASSEMBLY
TACHOMETER
POWER MOTOR
MOTOR - AMPLIDYNE

FIGURE B-1

Terminals are provided for picking off the voltage across a highly inductive resistor in the armature circuit

4. Amplifier Unit

The amplifier unit in general consists of two control amplifiers, thyrite resistor unit, field supply unit, and a heat exchanger and blower motor.

The thyrite resistor unit is connected across the control field of the amplidyne. Its function is to prevent excessive voltages from appearing across the field as a result of fast changes of current through the field inductance.

The field supply unit, as previously stated, furnishes a direct current to the field of the power motor, which current is maintained constant despite variations in field winding resistance because of temperature changes. A simplified sketch of the field supply unit is shown in Figure B-2.

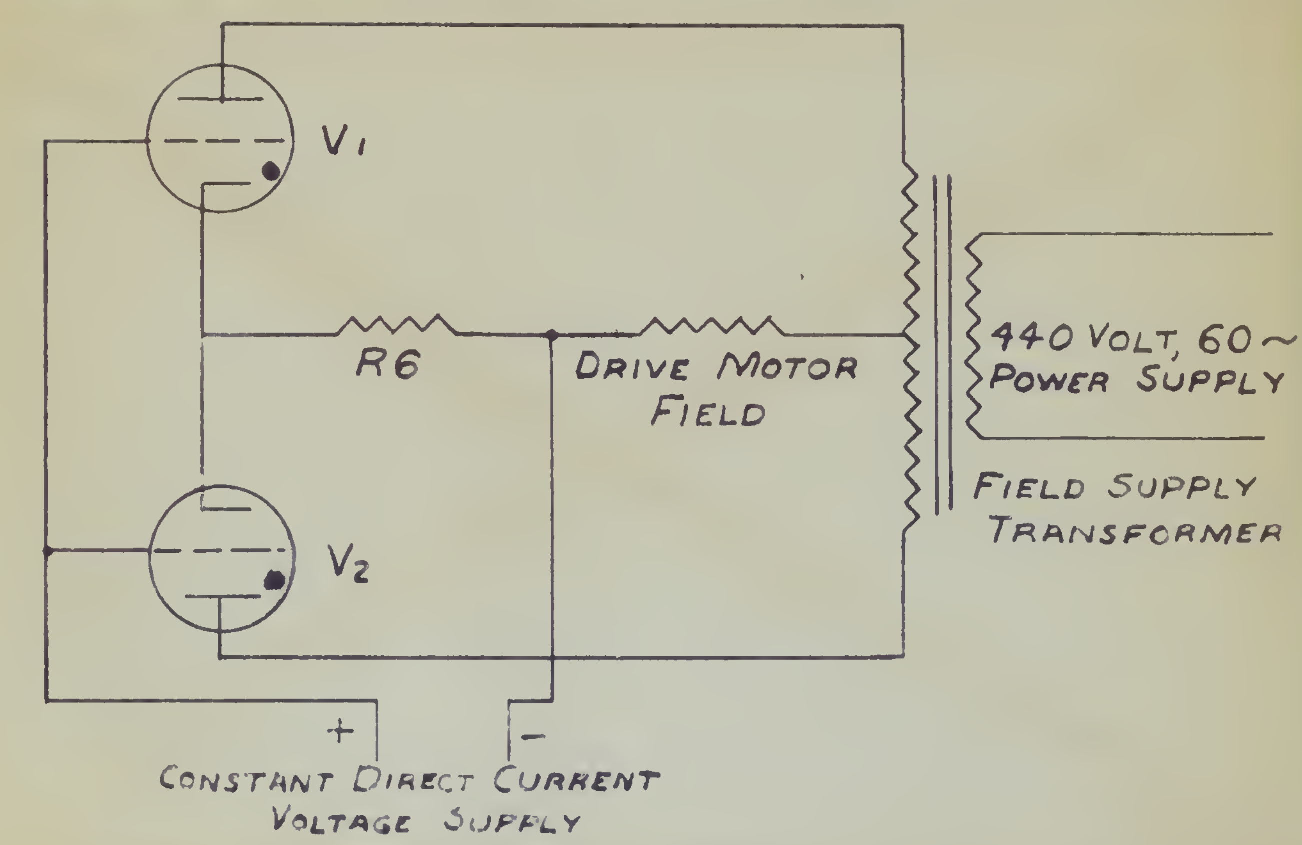


FIGURE B-2

Grid bias for thyratrons V_1 and V_2 consists of a positive voltage from a constant direct current supply and a negative voltage from the voltage drop across resistor R 6. The net bias is positive. The resistance value of R 6 is practically constant over all operating temperatures so that the voltage drop across R 6 is used to control the field current.

For example, assume the field resistance decreases. This would tend to increase the field current which would cause a greater drop across R 6, thereby decreasing the grid bias on tubes V_1 and V_2. As a result the output from these two tubes would decrease and the field current would return to its correct value.

The field supply unit also contains a thermal time relay, K1, which insures that the filaments of the thyratrons are heated to their operating temperature before the plate supply voltage is applied. In addition it also actuates contactor K7 in the control unit.

The heat exchanger and blower motor furnish outside air and a circulating system for cooling the electronic elements in the amplifier unit.

5. Reactor Assembly

The reactor assembly consists of three step-down transformers used to limit the current drawn from the 440 volt, 3 phase supply when the motor-amplidyne is started.

6. Control Unit

The control unit contains a series of magnetic controllers serving as safety devices for the motor-amplidyne and power motor.

In general, the 440 volt, 3 phase supply voltage goes directly

to a circuit breaker in the control unit. Contactor K1 connects the amplidyne's motor to the 440 volt supply through the starting reactor. Contactor K8 connects this motor directly across the 440 volt supply as soon as the timing relay has functioned. The timing relay, designated by TR on Figure B-1, is set for a five second operating period at the end of which time it picks up contactor K8.

Relay K7 is operated by the time delay relay in the field supply unit K1. This interconnection prevents operation of the amplidyne without power motor field excitation.

Overload relay, OL1, located in two phases of the 440 volt supply, is of the thermal-induction type and provides overload protection for the drive motor. This overload relay is reset by means of the electrical reset, RR1.

Overload relay, OL2, located in the power motor armature circuit, is a thermal type relay and provides overload protection to the amplidyne and power motor. This relay is reset by an electrical reset, RR2.

The local supply transformer furnishes 115 volt, 60 cycle supply for the magnetic controllers.

The field supply transformer furnishes the plate voltage for the thyratrons in the field supply unit.

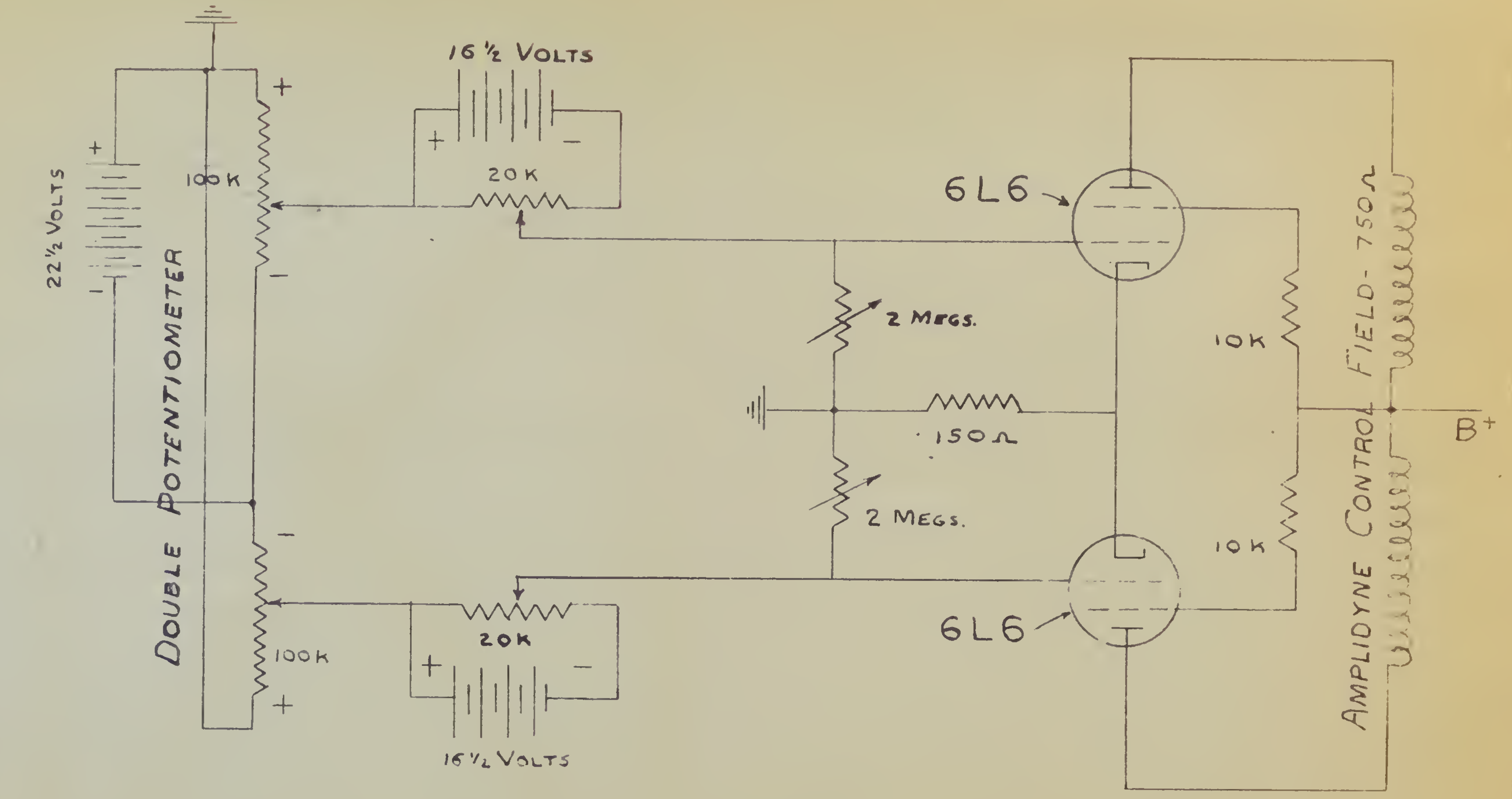
7. Power Supply

Since no 440 volt, 3 phase supply was available, it was necessary to use 4 to 1 step-up transformers. The transformers were connected $Y - \Delta$. Because of the heavy current drawn, even with the reactor in series, it was found impossible to start the motor on the 220 volt, 3 phase primary supply without fusing the line above its safe

operating limit. Therefore a triple pole double throw switch was put in the primary circuit of the transformers permitting the use of a 197 volt, 3 phase supply for starting and the 220 volt for running. Because of the poor regulation in the 197 volt supply the current never exceeded the line's safe capacity. When operating on the 220 volt supply, regulation was sufficient to reduce the secondary output line voltage to near 450 volts rather than the anticipated 520 volts as results at no load.

Instead of using either of the amplifiers supplied, one was constructed as indicated in Figure B-3. Its function was to amplify the signal to be applied to the amplidyne's control field. It consisted of two 6L6 tubes connected for push-pull operation with variable grid leak resistors for balancing purposes. The signals applied to the grids of these tubes were controlled by means of a double, 100 K, potentiometer connected across a 22-1/2 volt battery. Varying the input signals varied the output motor speed through control of the field difference current. Each grid circuit also contained a 16-1/2 volt battery across a 20 K potentiometer so that the operating point of the tube might be easily selected.

With this amplifier and the set-up as indicated in Figure B-1, tests were conducted for zero torque conditions. The amplidyne control field difference current was varied over the speed range of the motor in both clockwise and counter-clockwise directions. The speed of the motor for each value of difference current was observed by a hand tachometer in the lower speed levels and by means of an electric tachometer in the higher levels. The motor terminal voltage and the voltage developed by the tachometer generator were also observed. This data is presented



AMPLIFIER

FIGURE B-3

in Figures B-4, B-5, and B-6. For this run the zero difference current resulted from opposing 35 milliampere control currents.

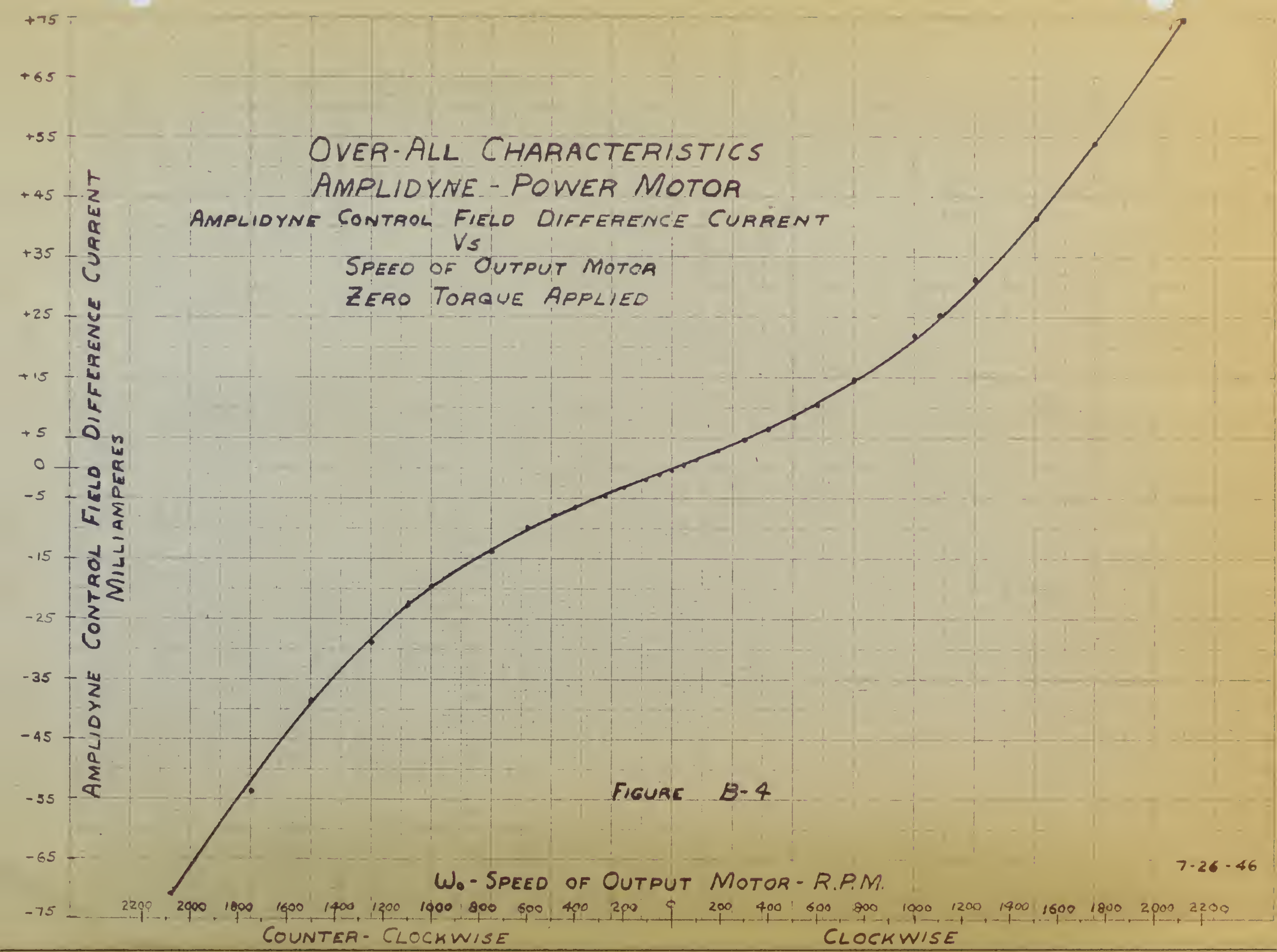
A standard type Prony brake was attached to the power motor as indicated in Figure B-7. The first run was of motor speed vs. control field difference current with no load other than the brake arm's friction drag applied. In this and subsequent tests, zero difference current results from the opposition of 65 milliampere currents. This data is presented in Figure 4-2.

Tests were conducted on the amplidyne-power motor combination under various torque conditions. A family of torque-speed curves for various constant control field difference currents was plotted from this data (Figure 4-1). Checks were made at the beginning and end of each run for the value of control field difference current necessary to stop the motor in both the clockwise and counter-clockwise direction. This value had a tendency to drift slightly during a run so that corrections to control field difference currents were applied, based on these zero readings.

In addition, it is to be recognized that the Prony brake is not accurate at low torque levels so that it is difficult to repeat data taken at 10 ft. lbs. or less. It is also in this range that hysteresis effects in the power motor are greatest, contributing to this low level in accuracy.

Other relationships for the amplidyne-power motor were obtained which might prove interesting to persons conducting a more complete study of the system. Curves exemplifying these relationships are enumerated below:

Figure B-8 Direct axis current vs. torque for various constant



OVER-ALL CHARACTERISTICS
AMPLIDYNE - POWER MOTOR
AMPLIDYNE CONTROL FIELD DIFFERENCE CURRENT
VS
SPEED OF OUTPUT MOTOR
ZERO TORQUE APPLIED
AMPLIDYNE CONTROL FIELD DIFFERENCE CURRENT
MILLIAMPERES
+75
+65
+55
+45
+35
+25
+15
+5
0
-5
-15
-25
-35
-45
-55
-65
-75
W_0 - SPEED OF OUTPUT MOTOR - R.P.M.
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
COUNTER- CLOCKWISE
CLOCKWISE
7-26-46

FIGURE B-4

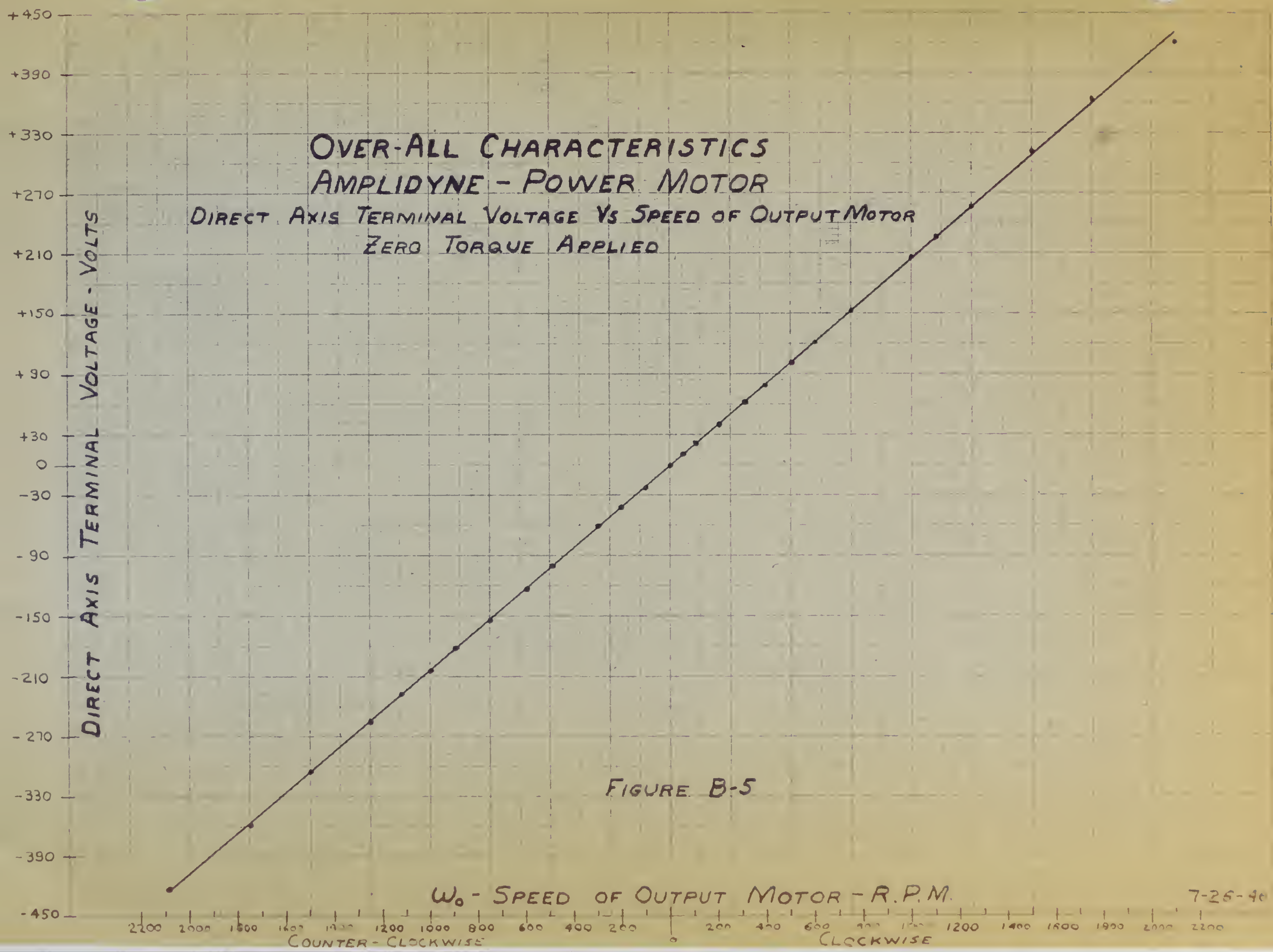

OVER-ALL CHARACTERISTICS
AMPLIDYNE - POWER MOTOR
DIRECT AXIS TERMINAL VOLTAGE VS SPEED OF OUTPUT MOTOR
ZERO TORQUE APPLIED
DIRECT AXIS TERMINAL VOLTAGE - VOLTS
+450
+390
+330
+270
+210
+150
+90
+30
0
-30
-90
-150
-210
-270
-330
-390
-450
ω_o - SPEED OF OUTPUT MOTOR - R.P.M.
2200 2000 1800 1200 1000 800 600 400 200
COUNTER-CLOCKWISE
0
200 400 600 1200 1400 1600 1800 2000 2200
CLOCKWISE
7-26-46

FIGURE B-5

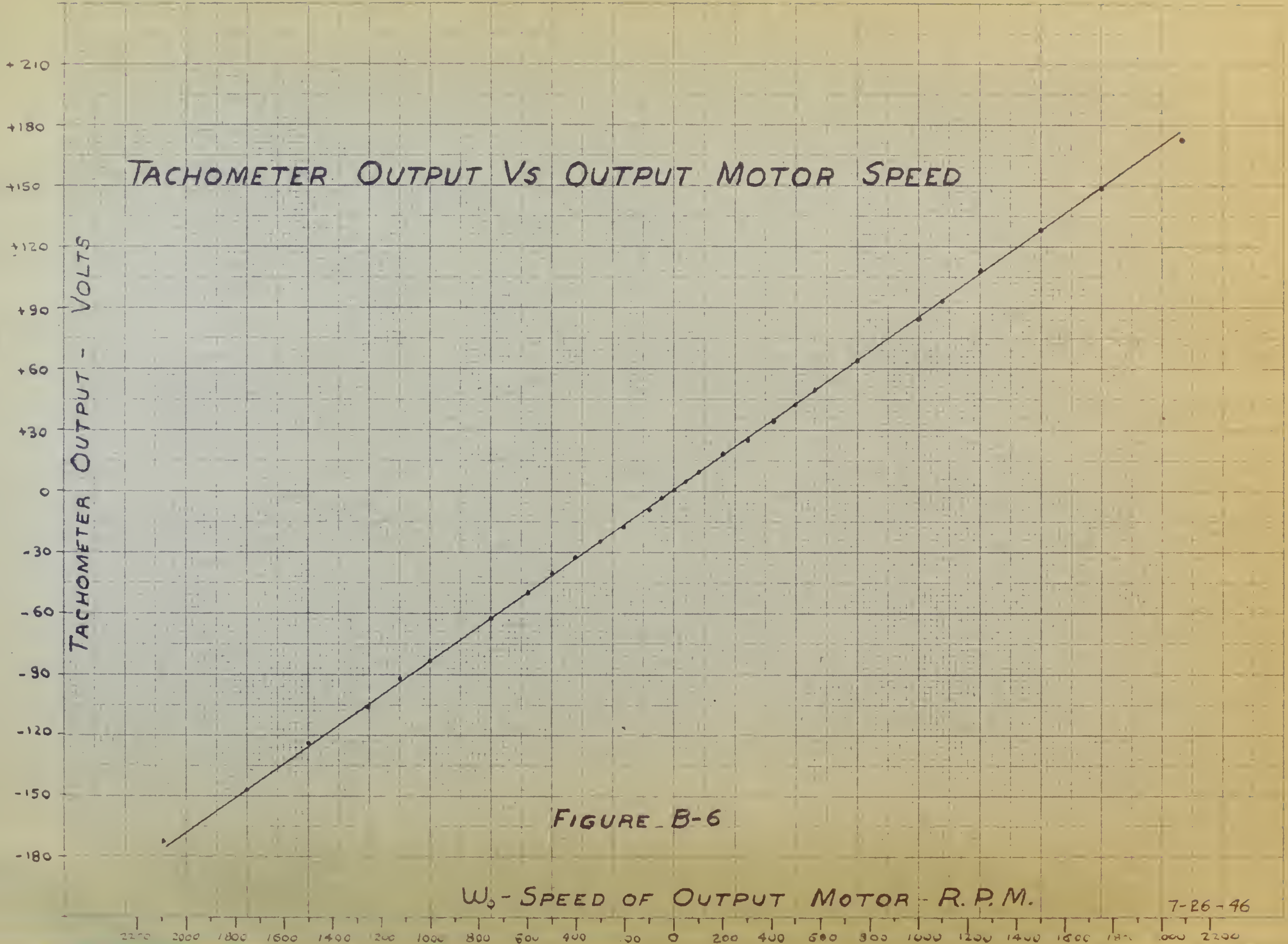


FIGURE B-6

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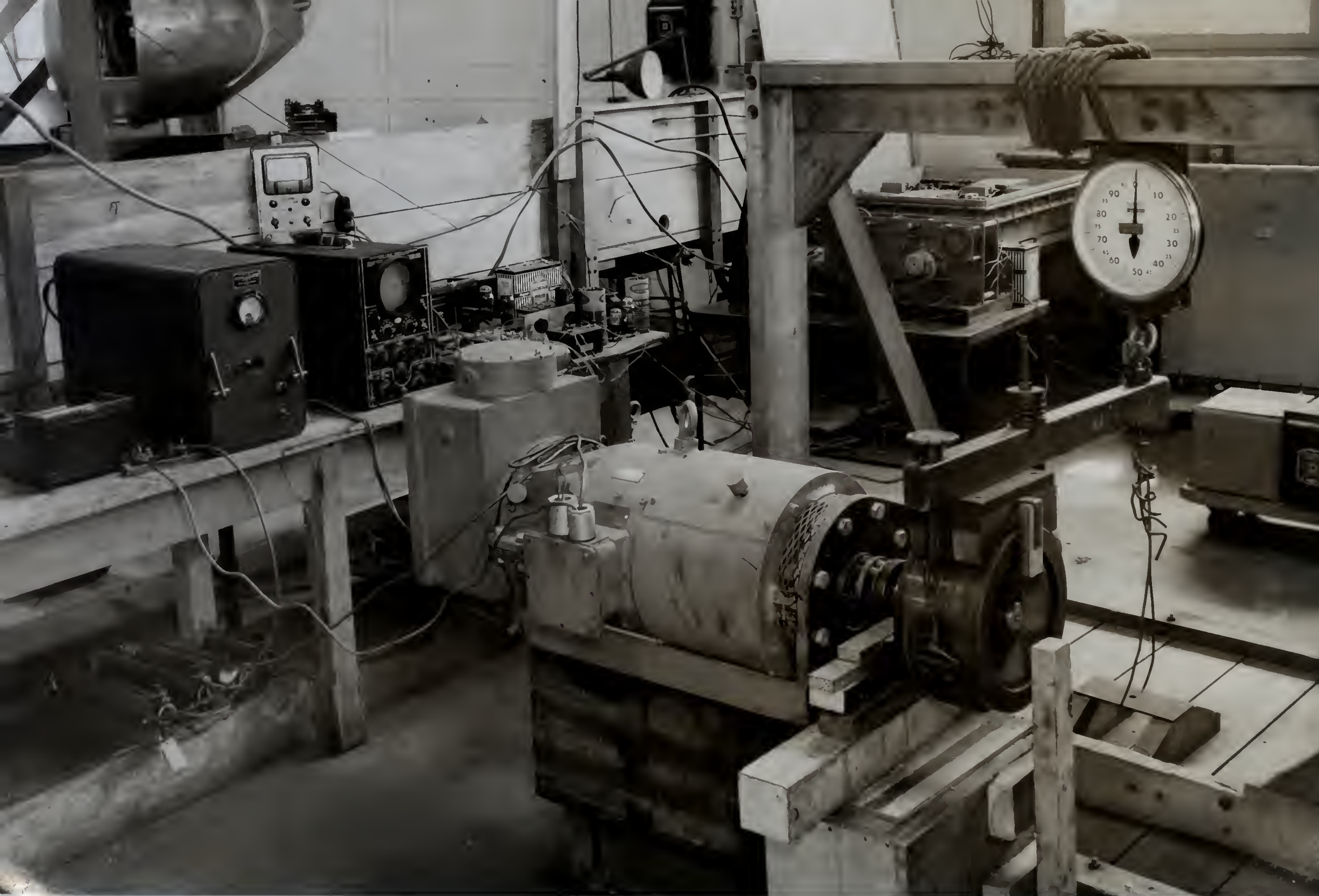


FIGURE 7A

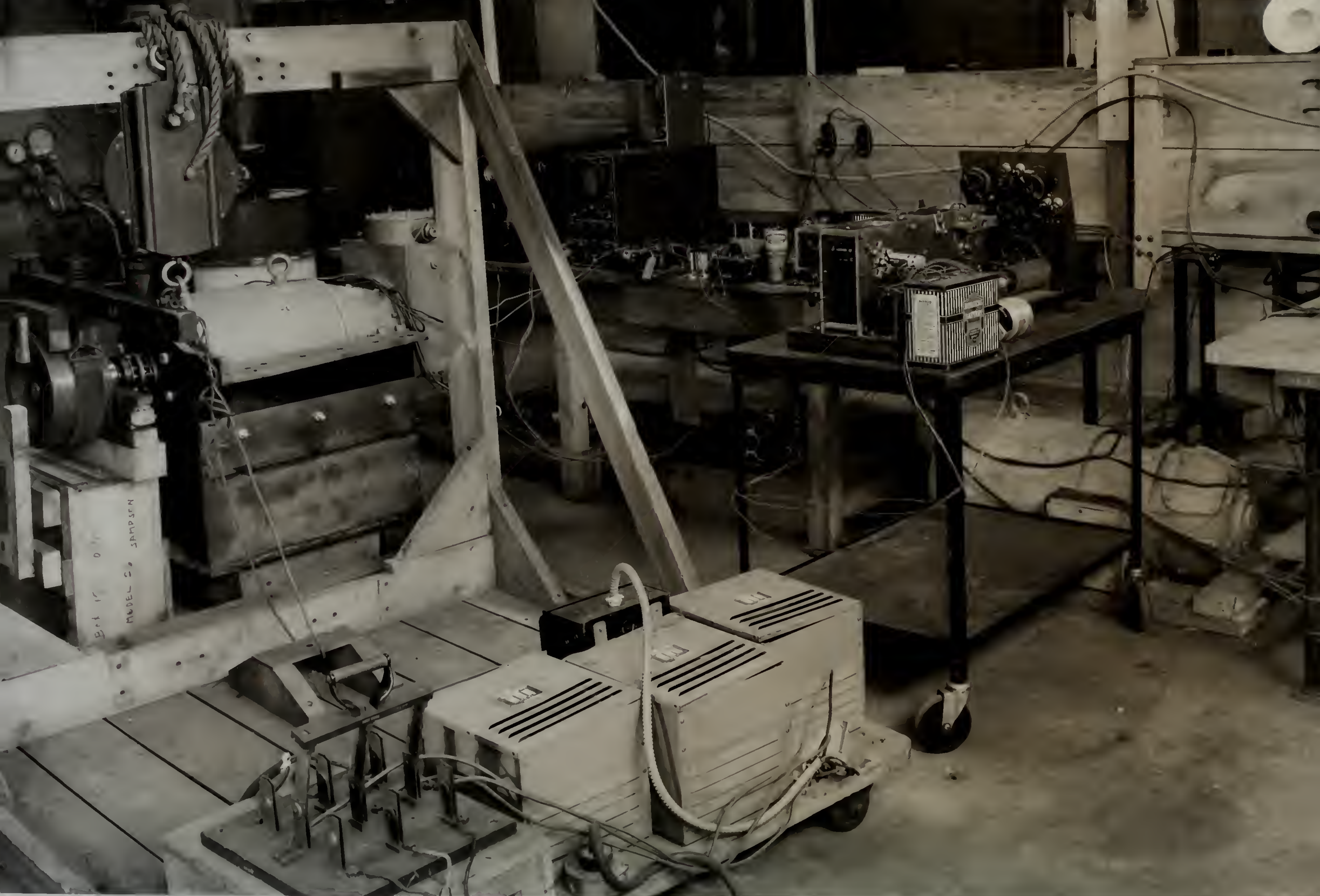


FIGURE 7B

values of control field difference current.

Figure B-9 Speed vs. motor terminal voltage for various constant torque levels.

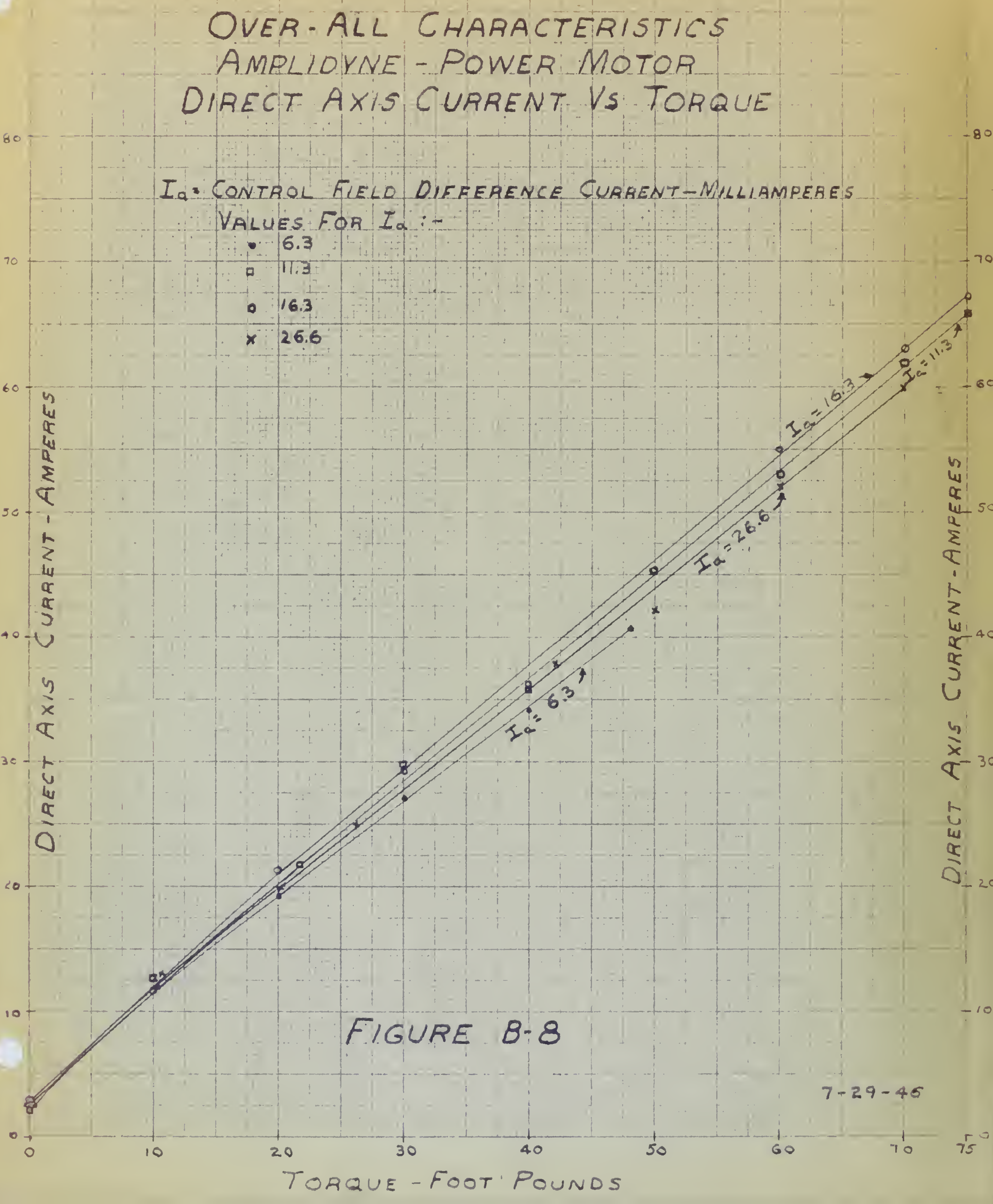
Figure B-10 Speed vs. control field difference current for various constant torque levels.

Of the curves resulting, the torque-speed relationships depicted in Figure 4-1 are the most important from an analytical point of view. A study of this figure indicates the following:

(a) The system is under-compounded. For low torque levels the speed falls off rapidly with the application of torque. Above the 15 ft. lb. torque level the torque-speed relationship becomes approximately linear.

(b) The curves indicate non-uniformity, i.e., the curves are unequally spaced for equal increments of control field difference current. Also at very low levels of control field difference current the shape of the curve differs from that obtained at higher levels and falls quickly to zero speed.

Appendix C discusses the methods used to overcome these undesirable conditions.



OVER-ALL CHARACTERISTICS
AMPLIDYNE - POWER MOTOR
DIRECT AXIS CURRENT VS TORQUE
I_a = CONTROL FIELD DIFFERENCE CURRENT—MILLIAMPERES
VALUES FOR I_a :-
6.3
11.3
16.3
26.6
I_a = 16.3
I_a = 11.3
I_a = 26.6
I_a = 6.3
DIRECT AXIS CURRENT - AMPERES
TORQUE - FOOT POUNDS
0
10
20
30
40
50
60
70
75
80
7-29-46

FIGURE B-8

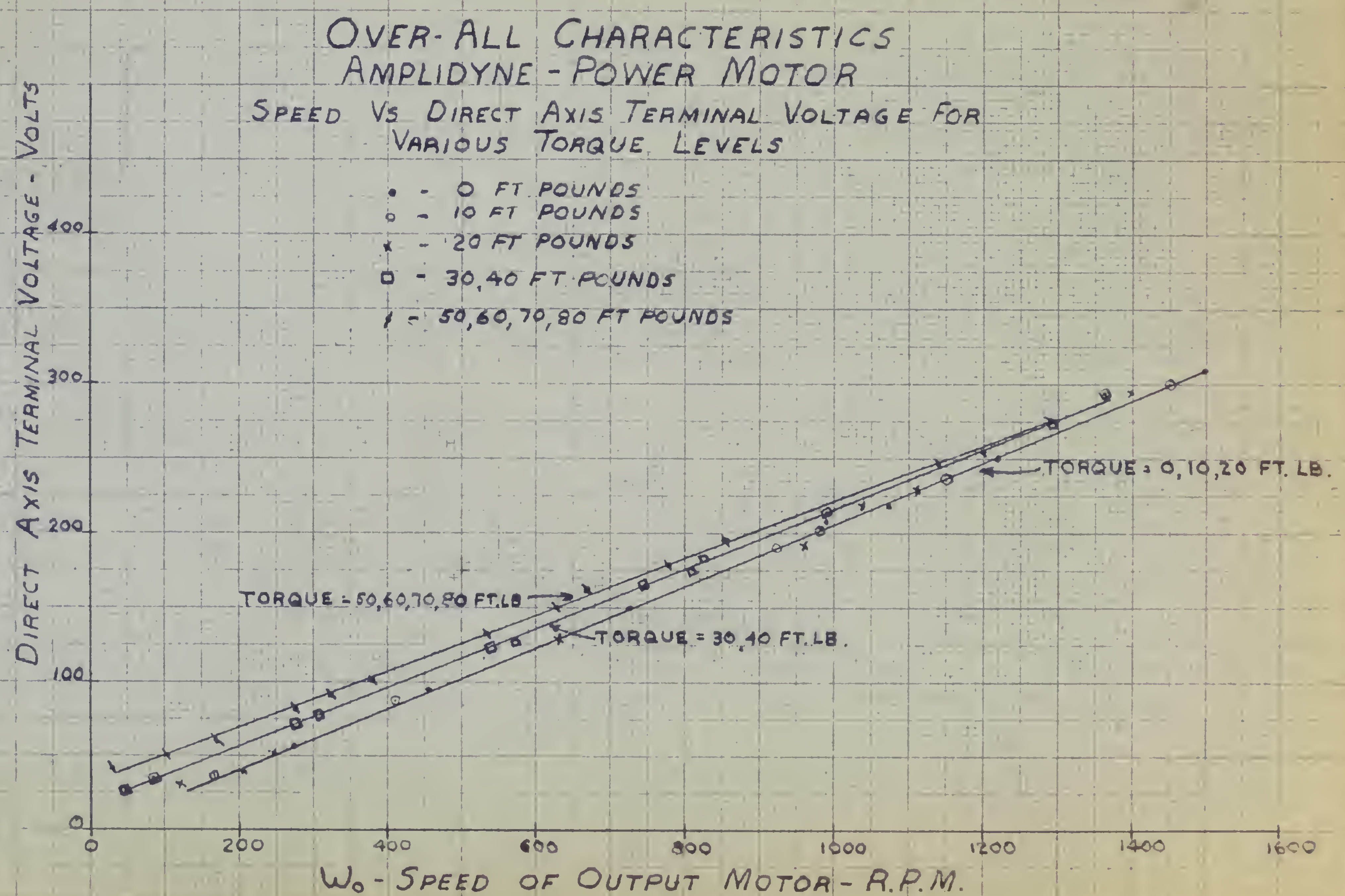


FIGURE B-9

7-29-46

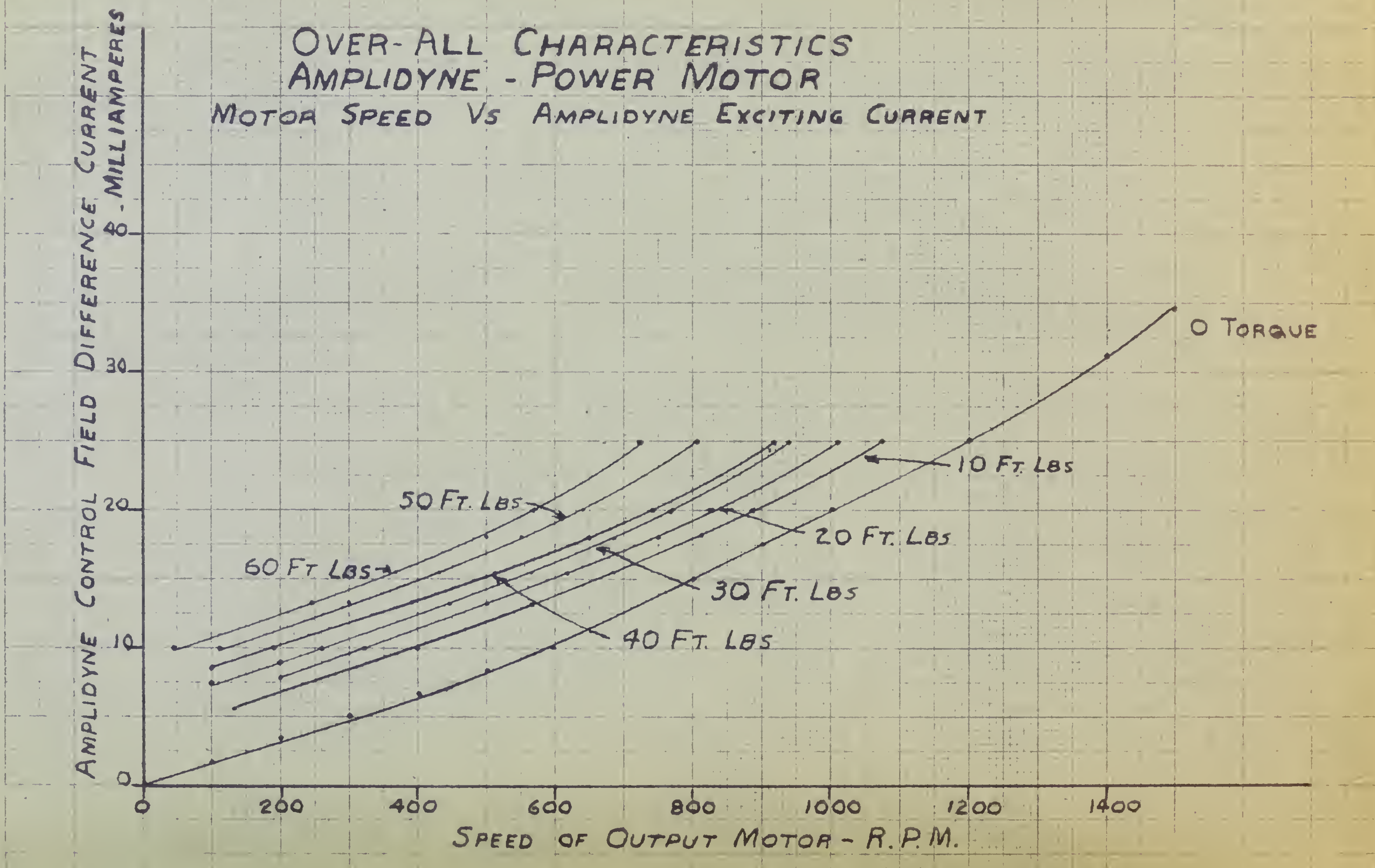


FIGURE B-10

7-29-46

APPENDIX C

TORQUE COMPENSATION

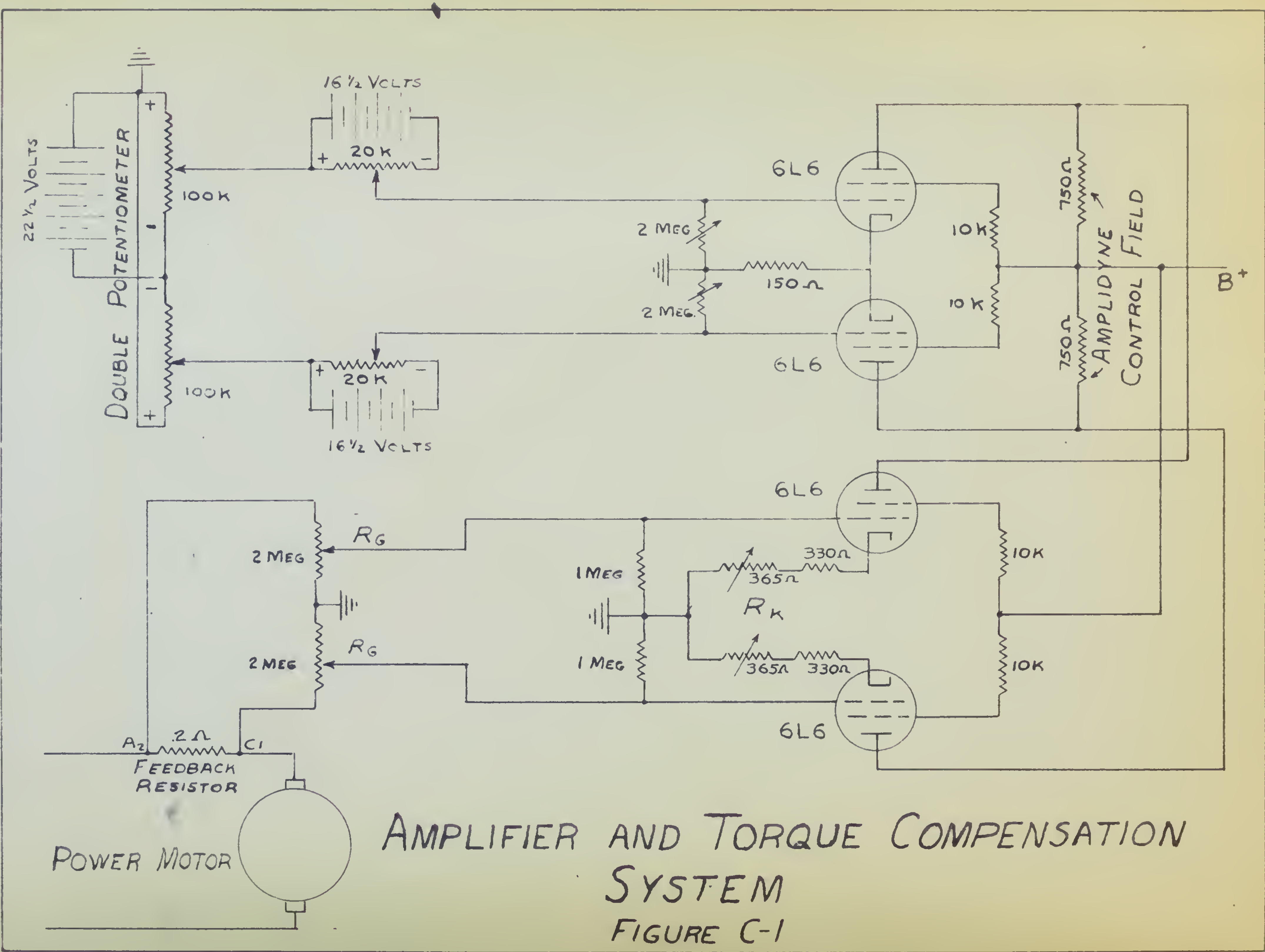
As indicated previously, it is not enough that the power drive be torque-compensated; it must also have a satisfactory frequency response. Of the requirements imposed by this last provision, one of the most important is the legislation against any resonant peaks occurring within the expected range of operating frequencies. Thus, though the subjects of torque compensation and frequency response are presented as two separate appendices, it must be kept in mind that the two problems were handled concurrently once initial experiment had indicated the possibility of obtaining satisfactory torque-speed curves.

It is the function of this appendix to report these initial experiments, to indicate the circuitry used, and to show the results obtained. Appendix D picks up the problem at this point and carries it through to the final result as presented in Figures 4-3 and 4-4, the ultimate torque-speed curves and frequency response. This division of the material is justified on the basis that, were torque compensation the only criterion, the results of the experimental work presented herein would have permitted satisfactory adjustment. All subsequent work was necessitated by the additional requirement of good frequency response.

As pointed out in Appendix B (see Figures 4-1, 4-2), two major faults were evident in the torque-speed curves of the amplidyne power drive when tested as a unit. These were:

(1) Poor speed regulation with torque load.

(2) Non-linearity in the speed versus exciting current relationship.



Amplifier and Torque Compensation System

Figure C-1

The first of these may be likened to the effect of an armature resistance. For a given terminal voltage (function of control difference current) an increase in armature current would increase the voltage drop due to this supposed armature resistance. Thus the back emf would be decreased, and, since back emf is proportional to speed, the speed would drop. This hypothetical resistance may be reduced in value or even made to appear negative by the use of positive feedback. This technique was the first employed. The circuit used was that shown in Figure C-1.

A description of basic operation follows. The resistor, A_2C_1, detected variations in motor armature current by the voltage drop across it. This voltage was applied through two potentiometers acting as a single resistor center-tapped to ground, thus producing equal but opposite signals on the potentiometers. Through the variable taps of the potentiometers the signals were carried to the grids of the tubes of the feedback amplifier whose plates were connected in parallel with those of the control amplifier. Since feedback was positive, increases in the grid swing increased the exciting current so that increased motor armature current resulted in increased power to the motor.

The system was simple but inadequate. The speed regulation with torque load was extremely poor over the low torque range (up to 15 ft. lbs.) and approximately linear thereafter till saturation effects took charge. This condition demanded that the gain of the feedback loop be initially high with subsequent reduction in gain to a constant value. The amplifier designed was approximately linear. Hence though excellent regulation resulted at high torques, initial speed drops were only partially eliminated. In addition, greater gain was demanded at low

speeds than at high, so that good adjustment at one level proved to be too much or too little at another. An analysis of this demanded variation in amplifier gain is shown in Table I. Figure C-2 shows a series of curves obtained while exploring the possibilities of this first circuit.

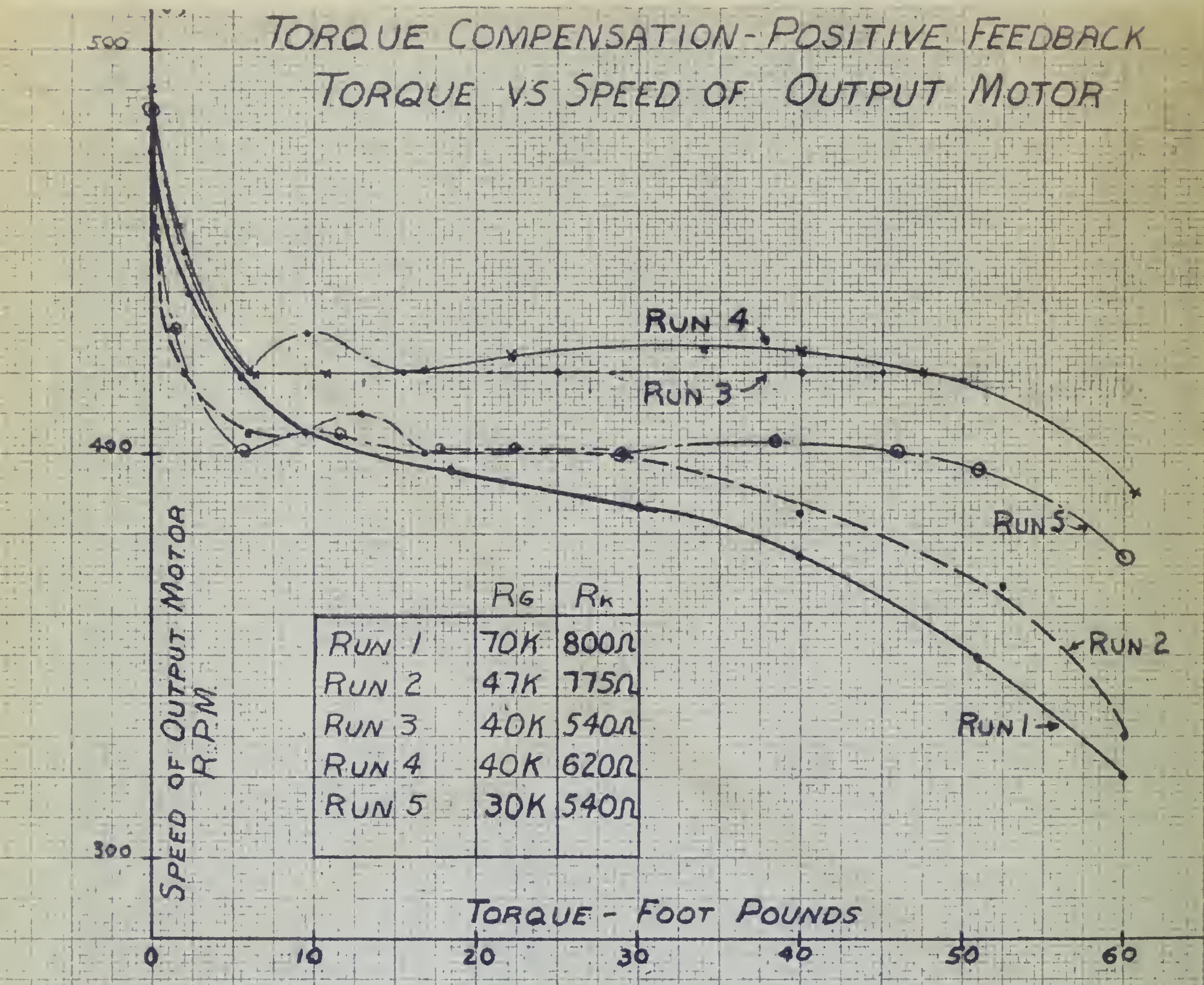


FIGURE C-2

TABLE I

ANALYSIS OF GAIN REQUIRED TO SECURE ZERO SLOPE TORQUE-SPEED CURVES UTILIZING POSITIVE FEEDBACK ONLY

CHANGE IN TORQUE (Δ T)	$\Delta I_a(5.5)$ to maintain constant speed	$\Delta I_d(10)$	$\Delta I_d(5.5)$ for ΔT	$\Delta I_d(10)$ for ΔT	$\Delta E_{fb}(5.5)$ for ΔI_d	$\Delta E_{fb}(10)$ for ΔI_d	Gain Required (5.5)	Gain Required (10)
	Milliamps.	Milliamps	Amperes	Amperes	Volts	Volts	$\frac{\text{Milliamps}}{\text{Volt}}$	$\frac{\text{Milliamps.}}{\text{Volt}}$
0 to 1 ft. lb.(1)	1.08	.95	.9	.7	.25	.2	4.32	4.25
1 ft. lb. to 2 ft. lbs. (1)	.42	.47	.9	1.3	.17	.23	2.47	2.04
2 ft. lb. to 3 ft. lbs. (1)	.30	.47	1.0	1.3	.18	.26	1.66	1.81
3 ft. lb. to 4 ft. lbs. (1)	.325	.31	1.2	1.3	.22	.24	1.48	1.29
4 ft. lbs. to 5 ft. lbs. (1)	.30	.38	1.0	1.0	.21	.20	1.43	1.90
5 ft. lbs. to 7.5 ft. lbs. (2.5)	.49	.58	2.3	3.0	.47	.55	1.04	1.05
7.5 ft. lbs. to 10 ft. lbs. (2.5)	.48	.36	2.1	2.3	.48	.35	1.00	1.03
10 ft. lbs. to 20 ft. lbs. (10)	1.3	1.1	8.4	7.3	1.42	1.41	.915	.78

5.5 and 10 refer to control field difference current at which each test was run.

I_d is the motor armature current.

I_a is the control field difference current.

E_{fb} is the positive feedback voltage.

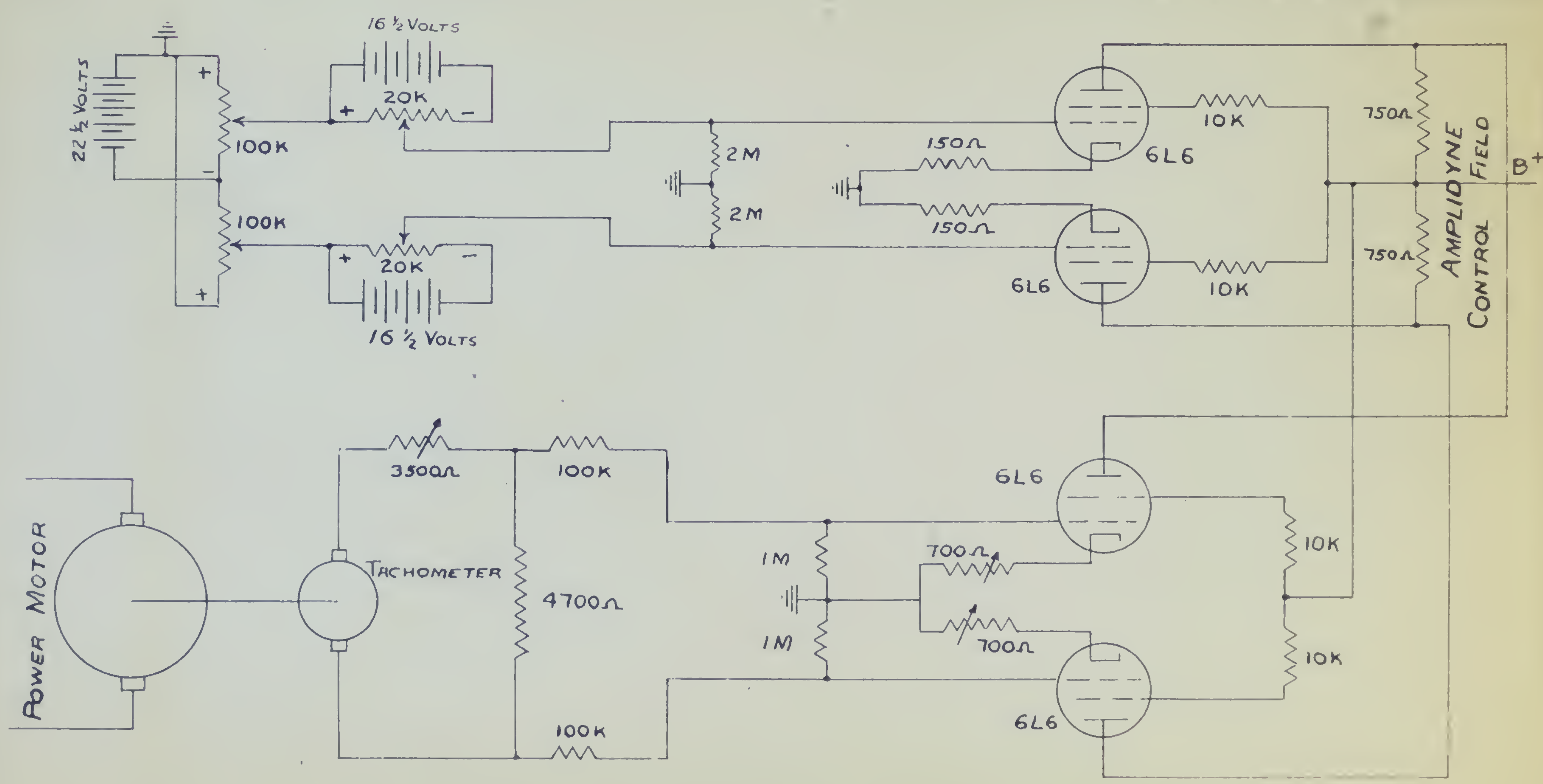
No ready means of obtaining an automatic variable gain from the feedback amplifier suggested itself. It was apparent that an additional control was needed. Since the function to be controlled was speed, it was logical to use speed feedback in some way to obtain this other correction. Also it was anticipated that the second major fault of the original curves would be eliminated or at least palliated.

The synchro unit (described in Appendix B) contains a tachometer. By applying negative tachometric feedback to the grids of the feedback amplifier an approximate speed servo loop resulted. (See Figure C-3). For any given speed, an increase in torque tended to decrease the speed. The initial speed drop resulted in a smaller signal from the tachometer thus allowing the control input signal to have greater effect. This increased the exciting current and power to the motor. A new speed balance was reached only slightly less than the original speed. It was evident that the positive feedback would augment this action by decreasing the amount of regulation the tachometric feedback had to overcome. It seemed a question of balancing the two corrective actions so that their net effect would be satisfactory but their coupling small.

Having sufficient data on the positive feedback system alone, the tachometric feedback system was tested separately. Results are shown in Figure C-4.

This completed the independent work on torque compensation. Analysis of the resultant data was made and certain conclusions drawn before proceeding to the combined problem covered in Appendix D. The conclusions reached were:

1. Positive current feedback and negative tachometric feedback



TORQUE COMPENSATION CIRCUIT FOR TACHOMETRIC FEEDBACK

FIGURE C-3

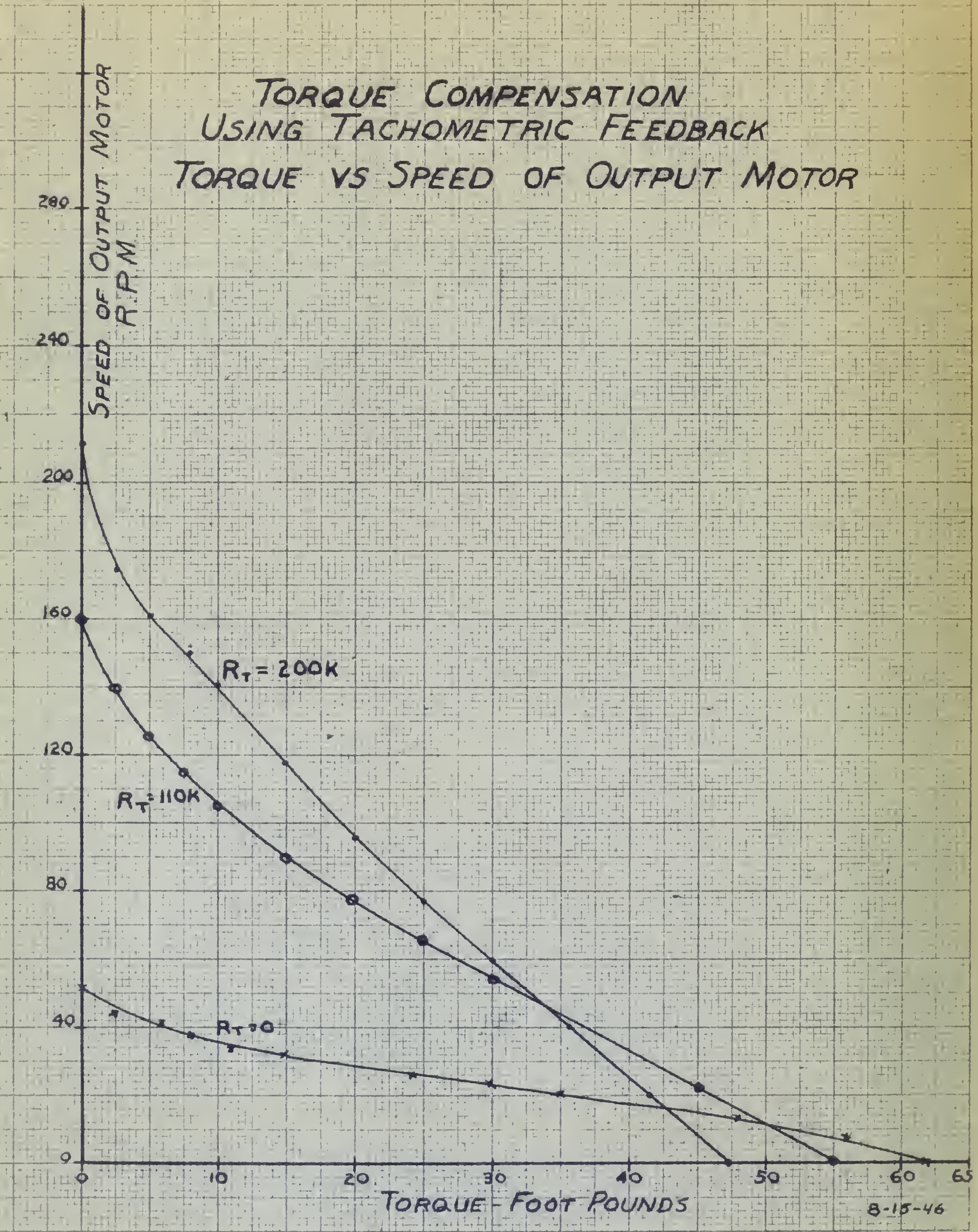

TORQUE COMPENSATION
USING TACHOMETRIC FEEDBACK
TORQUE VS SPEED OF OUTPUT MOTOR
SPEED OF OUTPUT MOTOR
R.P.M.
280
240
200
160
120
80
40
0
$R_T = 200K$
$R_T = 110K$
$R_T = 0$
0
10
20
30
40
50
60
65
TORQUE - FOOT POUNDS
8-15-46

FIGURE C-4

would be closely coupled if the circuits previously used (Figures C-1, C-3) were combined directly.

2. To reduce this coupling, through resistive networks, attenuation of the feedback signals would result. Present gain levels did not permit this without destroying the feedback effects and it was desired to keep electronic elements to a minimum thus decreeing against amplifiers for each feedback signal.

3. It seemed logical to apply the positive feedback to the grids of the feedback amplifier as was previously done but to shift the tachometric feedback to the control signal amplifier thus decoupling the feedbacks. This also would simplify installation of corrective networks for frequency work.

4. This change might lead to a need for greater amplification of the control signal, but this signal is generally amplified so that a stiffer requirement should pose no problem.

5. Both feedbacks produced better torque-speed curves. Positive feedback could provide rising characteristics at high torque levels, and very little regulation at low torques.

6. Tachometric feedback not only attempted to prevent slow-downs but also legislated partially against speeding up.

7. A combination of excessive positive feedback (rising characteristic) with considerable tachometric feedback to provide stability and to limit the speed increase seemed sound and practical.

8. If necessary, diode clampers or relays could be used in the positive feedback path to restrict this feedback to a definite limit,

the bias of the clamper or the coil of the relay to be so adjusted as to function when the torque had reached a certain value. (Basically as previously shown in Figure B-8 motor armature current is directly proportional to torque.)

9. It might prove desirable to use a separate resistive element in the motor armature circuit to provide voltage for use as positive feedback. The resistive element supplied in the motor is highly inductive.

APPENDIX D

FREQUENCY RESPONSE

The equipment used to obtain all frequency response test data consisted of a sine drive to provide the forcing function and a five inch Dumont cathode ray oscilloscope to view the input and output presented as a Lissajous figure.

The sine drive had been built previously for use in such work, and is a standard item in the Servomechanisms Laboratory at M. I. T. Briefly, it is an AC electric motor with variac speed control driving a rack in reciprocating linear motion by means of a Scotch yoke. The rack meshes with a pinion which serves as the first gear in a long train driving ultimately the shaft to which the double control potentiometer was attached. The linear motion of the rack was converted into rotary motion which, by the principle of the Scotch yoke, was purely sinusoidal except for machining inaccuracies and backlash in the gearing. The potentiometer thus mounted produced two sine waves 180° out of phase to serve as control signals. Later work necessitated the mounting of a second such potentiometer to provide an input reference signal for the oscilloscope.

The input reference signal for the oscilloscope, coming from a potentiometer energized by battery, was too weak to be used directly on the plates. When tried on the amplifier of the oscilloscope, the signal was found to be too jittery, producing a line of varying intensity and length rather than a spot. Any condenser sufficient to remove this pickup and "hash" served to mask all low frequency change in the signal.

In consequence a simple amplifier was built for this input signal (Figure D-1). The output of this amplifier was attached directly to the X-axis plates with .1 microfarad shunting condensers to the case.

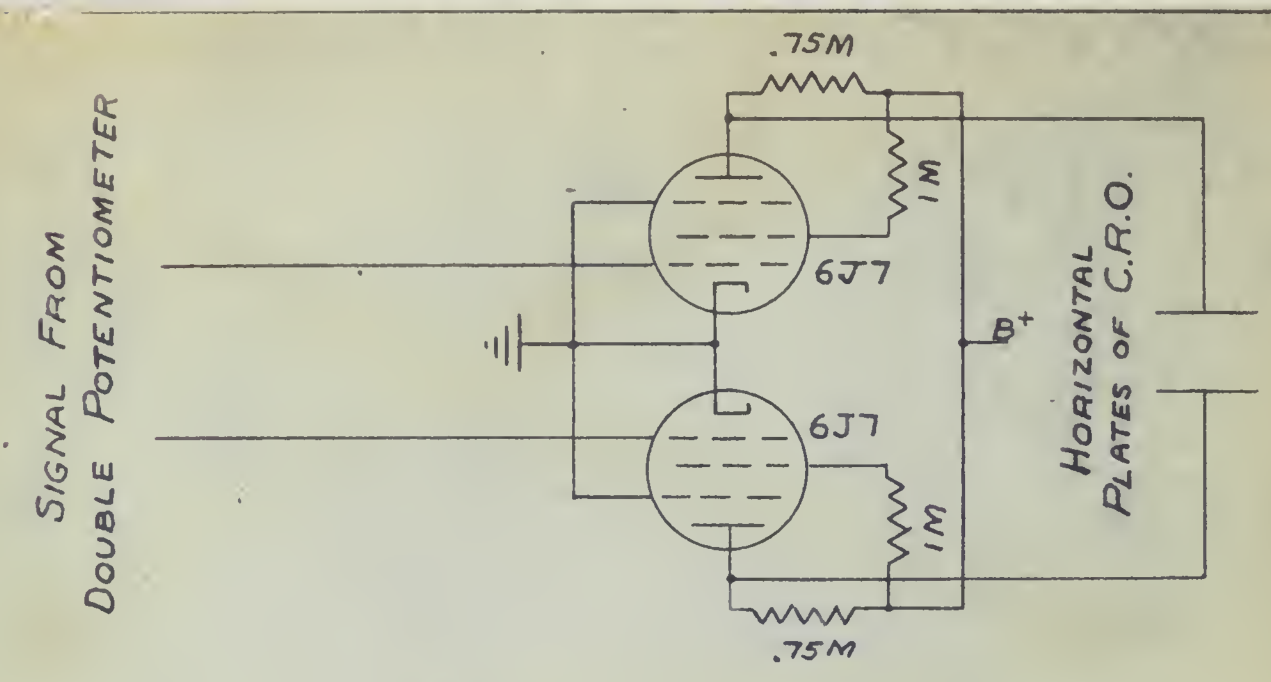


FIGURE D-1

The output reference signal was taken from the tachometer in the synchro unit. The tachometer was well made with sufficient commutator bars so as to have no discernible ripple in its output. Pickup in this generator was negligible. The plot of tachometer output voltage versus speed (Figure B-6) proved this voltage to be an excellent representation of speed even down to zero speed levels. The magnitude of the tachometer output voltage was such as to permit it to be applied directly to the Y-axis plates of the CRO. No shunting condensers were employed.

The first run provided data for the plotting of the frequency response of the amplidyne motor without feedback. (Figure 4-4).

This data served as a framework against which to measure improve-

ment in the frequency response and from which to deduce certain aims. The improvement desired consisted, first, of keeping the phase shift as small as possible with the hope of reaching 90° at a frequency greater than one cycle per second, and second, of keeping the gain at unity as far out as possible. In this connection it was deemed best to accept a slight resonant peak in exchange for a higher resonant frequency and consequent improved phase shift.

While discussing frequency response, it is appropriate to consider transient response--another form of the same data. During the work of torque compensation discussed in Appendix C it had been noted that the transient action of the system responding to a step function of torque had been very poor with positive feedback alone, fair considering the motor without any feedback, and faster with tachometric feedback. It was the additional aim, therefore, during these frequency response tests to obtain as fast a transient response as possible with the desire that the system be near critically damped but favoring underdamping. This decision was premised on the fact that with slightly less than critical damping there would be one or two small speed oscillations but the noticeable transient action would be over faster than if the system were critically damped or overdamped. These oscillations had to be kept small so that their effect on the personnel riding the gun, through the introduction of large accelerations, would not be disastrous. This fitted in well with the previous reasoning on the desired improvement of the frequency response.

Much experimental work followed. The resistive element in the motor was replaced by an external pure resistance as the source of

positive feedback. By increasing the size of this resistor it was possible to obtain rising torque-speed characteristics for the motor with higher values of cathode resistance in the feedback amplifier than had previously been possible. This provided a more linear output from this amplifier with change in grid signal. Tachometric feedback was applied to the grids of the control signal amplifier. By virtue of this change less attenuation of the signal was necessary providing increased speed control. The values of attenuating resistors in the two feedback networks and the settings of the cathode resistors in the feedback amplifier were determined empirically.

With torque-speed curves resulting from test data showing fair regulation, attention was turned to corrective networks in the feedback paths to enhance frequency response. A lead network in the tachometric feedback path would help speed up response and increase gain by making the system responsive to acceleration demands. In addition it would decrease the time lag in the system and aid in holding down phase shift. Since all frequencies involved were in the vicinity of 1 cps, a time constant in this lead network near .15 seconds was judged satisfactory. Using a decade box capacitance variable from 1 to 10 microfarads, the resistive elements in the tachometric feedback path were stepped up proportionately to the magnitude of hundreds of kilohms to provide circuit time constants near this value. Since the positive feedback path led to instability, an integral network was placed in its path to smooth out rapid fluctuations in voltage. This permitted the use of greater positive feedback, but slowed the overall system a little and increased the phase shift. Attenuating resistors on the order of a

megohm were introduced. Time constants much smaller than in the lead network were anticipated so that the changes in the positive feedback voltage would not be completely suppressed. This led to selection of a decade box capacitance variable from .01 microfarad to 1.1 microfarad in .01 microfarad steps.

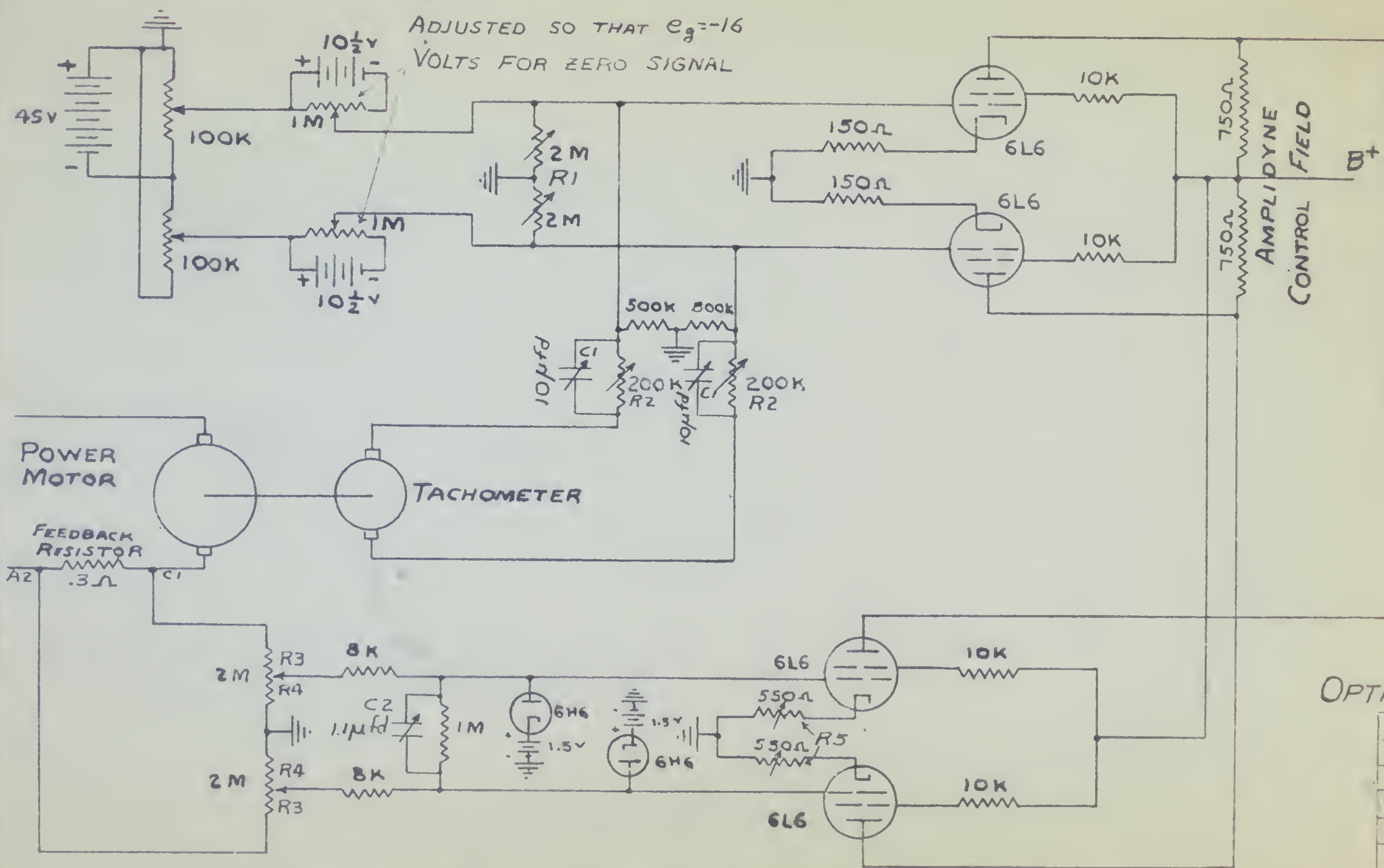
Although it was considered possible to improve overall response through the use of a filter circuit in the control signal input, no such tests were conducted. The reasoning governing this decision was that any such network was actually external to the amplidyne-motor combination. As such it was an element in the rate servo loop using this power drive. Since it was already planned that the synthesis of such networks in the overall loop would be done graphically in Chapter 5, using the decibel-log frequency technique, there was nothing to be gained by duplication of effort.

To select the final values of time constants in each corrective network the amplidyne was energized with sufficient signal to give a low speed output at the motor. By applying a step function of torque the condenser values were adjusted to give a fast slightly underdamped transient response. The control signal was then changed to a sinusoidal input at the frequency which corresponded to the 90° phase shift point of the uncompensated amplidyne-motor. The condenser values were again adjusted to reduce this phase shift to a minimum. This setting was then tested by increasing the frequency to note the magnitude of the resonant peak and the new frequency at which 90° phase shift occurred. It became necessary to accept slightly poorer phase shift adjustment in order to keep the resonant gain peak below 1.3. A compromise was

then reached between the observed transient response and the measured frequency response in the vicinity of resonance by averaging the values of condenser settings corresponding to each optimum condition. Since the condenser values were almost identical, subsequent check showed little change in performance.

The attenuation in the feedback paths was varied to give close speed regulation. The criterion selected was that speed change should be no more than $\pm$ 10% of no load speed as load increased from 0 to 60 ft. lbs. Sixty ft. lbs. was considered maximum since at values slightly higher than this the overload relay in the motor armature circuit tripped. It was found possible to meet this condition down to no load speeds as low as 200 rpm's. Below this the best that could be done was to restrict speed variations to $\pm$ 20 rpm's. In consequence below about 30 rpm's the motor performance was unreliable and stalled repeatedly with a load of 8 to 10 ft. lbs. To obtain this performance two diode clampers in parallel with 1.5 volts above ground cathode bias were inserted in the positive feedback channel to limit the effect of this feedback at high torques (above about 10 ft. lbs.). After the corrective networks were adjusted as described in the last paragraph it was found that the best frequency response resulted from this optimum adjustment for torque-speed characteristic. Thus no compromise was required between these two important performance criteria. Figure D-2 has been included to show graphically the net improvement in the torque-speed curves and the separate effect of each type of feedback.

With the circuit adjusted as shown in Figure D-3, final data for



OPTIMUM SETTINGS

R1	2 MEG
R2	110K
R3	70K
R4	1.93 MEG
R5	510Ω
C1	2 μfd
C2	0.3 μfd

TORQUE COMPENSATION CIRCUIT WITH POSITIVE CURRENT AND TACHOMETER FEEDBACK

FIGURE D-3

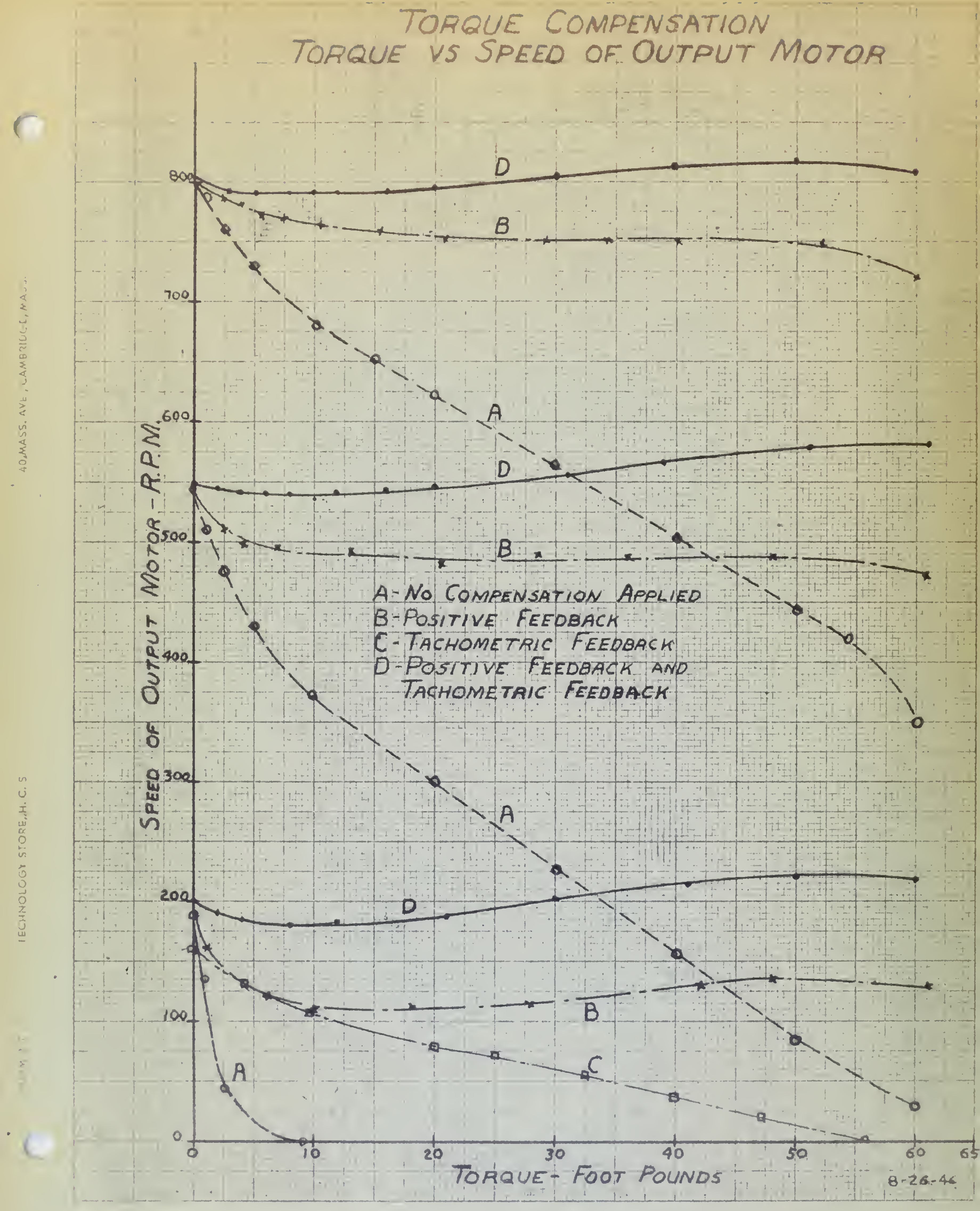

TORQUE COMPENSATION
TORQUE VS SPEED OF OUTPUT MOTOR
SPEED OF OUTPUT MOTOR - R.P.M.
TORQUE - FOOT POUNDS
A - NO COMPENSATION APPLIED
B - POSITIVE FEEDBACK
C - TACHOMETRIC FEEDBACK
D - POSITIVE FEEDBACK AND TACHOMETRIC FEEDBACK
A
B
C
D
0
100
200
300
400
500
600
700
800
10
20
30
40
50
60
65
8-26-46
TECHNOLOGY STORE, H. C. S
40 MASS. AVE., CAMBRIDGE, MASS.

FIGURE D-2

torque-speed curves and frequency response were obtained. These curves are presented in Figures 4-3 and 4-4 respectively. It is interesting to note that, as estimated in Chapter 3, the final frequency response of the amplidyne-motor may be closely approximated by a transfer function of the form $\frac{K}{T^2 s^2 + 2\zeta Ts + 1}$ where T = .133 second and ζ = .45.

This completed the experimental work for the thesis.

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